1. (5 points) The position of an object along a line is given by the function $s(t) = 2t^3 - 5t^2 + 3$, where t is measured in seconds and s in feet.

   (a) What is the average velocity over the interval $[1, 3]$?

   Solution:

   $$s_{\text{avg}} = \frac{s(3) - s(1)}{3 - 1} = 6$$

   (b) What is the (instantaneous) velocity at time t?

   Solution:

   $$v(t) = s'(t) = 6t^2 - 10t$$

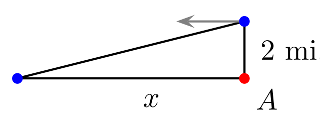
   (c) For $t \geq 0$, when is the object moving in the positive direction?

   Solution:

   We need solve the inequality $s'(t) > 0$. This is easily seen to be true when $t > 5/3$.

2. (5 points) An airplane is approaching an airport at a constant altitude of 2 miles. Let θ be the angle of elevation from the airport to the plane. If the speed of the plane is 200 miles per hour, how fast is θ increasing when the airplane is 10 miles away from the airport (see sketch)?

   Solution:

   Let θ be the angle of elevation from the airport to the plane and let x be the distance from a point A to the airport (see the sketch). Then the quantities θ, x, and 2 are related by the equation $\tan\theta = 2/x$ Differentiating both sides with respect to time yields

   $$\sec^2\theta \frac{d\theta}{dt} = \frac{-2}{x^2}\frac{dx}{dt}$$

   It follows that

   $$\frac{d\theta}{dt} = \frac{-2\cos^2\theta}{x^2}\frac{dx}{dt}$$

   Notice that $\cos\theta = 10/\sqrt{104}$ when $x = 10$. Thus

   $$\left.\frac{d\theta}{dt}\right|_{x=10} = \frac{-2(100)}{10^2(104)}(-200) = \frac{50}{13} \text{ radians per hour}$$

   Why is $dx/dt < 0$?