1. (4 points) Find $f'(x)$ if $f(x) = \cos\sqrt{1+x^2}$.

Solution:

We employ the Chain Rule.

$$\begin{aligned} f'(x) &= \left(-\sin\sqrt{1+x^2}\right)\cdot D_x\left(\sqrt{1+x^2}\right) \\ &= \left(-\sin\sqrt{1+x^2}\right)\left(\frac{1}{2\sqrt{1+x^2}}\right)\cdot D_x\left(1+x^2\right) \\ &= \left(-\sin\sqrt{1+x^2}\right)\left(\frac{1}{2\sqrt{1+x^2}}\right)(2x) \end{aligned}$$

2. (3 points) Suppose that f and g and their derivatives with respect to x have values at $x = 2$ and $x = 5$ given the table below. Let $h = f \circ g$. Find $h'(2)$. In other words, use the following table to find $(f \circ g)'(2)$. *Note: Not all values are needed.*

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	1	5	−3	−6
5	0	7	4	−8

Solution:

According to the Chain Rule, $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$. It follows that

$$h'(2) = (f \circ g)'(2) = f'(g(2)) \cdot g'(2) = f'(5) \cdot g'(2) = 4 \times (-6)$$

3. (3 points) Observe that the point $P = (3, 1)$ lies on the curve $x^2 - 4xy + 9y^2 = 6$. Find the slope of the line tangent to the curve at P. In other words, find $\left.\frac{dy}{dx}\right|_{(x,y)=(3,1)}$.

Solution:

Differentiating both sides of the given equation yields

$$2x - 4\left(y + x\frac{dy}{dx}\right) + 18y\frac{dy}{dx} = 0$$

It follows that

$$\frac{dy}{dx} = \frac{4y - 2x}{18y - 4x}$$

Hence

$$\left.\frac{dy}{dx}\right|_{(x,y)=(3,1)} = \frac{4-6}{18-12} = \frac{-1}{3}$$