1. (3 points) Find $g'(x)$ if $g(x) = \dfrac{5x - 3x^2}{4x^2 + 1}$. *You may use any method to find the derivative.*

Solution:

We use the Quotient Rule.

$$\begin{aligned} g'(x) &= \frac{D_x(5x - 3x^2)(4x^2 + 1) - (5x - 3x^2)D_x(4x^2 + 1)}{(4x^2 + 1)^2} \\ &= \frac{(5 - 6x)(4x^2 + 1) - (5x - 3x^2)(8x)}{(4x^2 + 1)^2} \\ &= \frac{5 - 6x - 20x^2}{(4x^2 + 1)^2} \end{aligned}$$

2. (3 points) Find equation of the tangent line to the curve $p(x) = x^2 \sin x$ at $x = \pi/2$.

Solution:

$$p'(x) = 2x \sin x + x^2 \cos x$$

It follows that the slope of the tangent line at $x = \pi/2$ is

$$\begin{aligned} m &= p'(\pi/2) \\ &= \pi \sin\frac{\pi}{2} + \left(\frac{\pi}{2}\right)^2 \cos\frac{\pi}{2} \\ &= \pi + 0 \end{aligned}$$

So the equation of the tangent line is given by

$$y - \frac{\pi^2}{4} = \pi\left(x - \frac{\pi}{2}\right)$$

3. (4 points) Let $f(x) = \sqrt{1+2x}$. Use the limit definition to find $f'(x)$. *Show your steps.*

Solution:

$$\begin{aligned}
f'(x) &= \lim_{z\to x} \frac{f(z)-f(x)}{z-x} \\
&= \lim_{z\to x} \frac{\sqrt{1+2z}-\sqrt{1+2x}}{z-x} \\
&= \lim_{z\to x} \frac{\sqrt{1+2z}-\sqrt{1+2x}}{z-x}\,\frac{\sqrt{1+2z}+\sqrt{1+2x}}{\sqrt{1+2z}+\sqrt{1+2x}} \\
&= \lim_{z\to x} \frac{(1+2z)-(1+2x)}{(z-x)\left(\sqrt{1+2z}+\sqrt{1+2x}\right)} \\
&= 2\lim_{z\to x} \frac{z-x}{(z-x)\left(\sqrt{1+2z}+\sqrt{1+2x}\right)} \\
&= 2\lim_{z\to x} \frac{1}{\left(\sqrt{1+2z}+\sqrt{1+2x}\right)} \\
&= \frac{2}{2\sqrt{1+2x}}
\end{aligned}$$