1. (3 points) Evaluate the following limit. *Show your work.*

$$\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16}$$

Solution:

$$\begin{aligned}\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} &= \lim_{x\to 16} \frac{\sqrt{x}-4}{(\sqrt{x}+4)(\sqrt{x}-4)} \\ &= \lim_{x\to 16} \frac{1}{\sqrt{x}+4} \\ &= 1/8\end{aligned}$$

2. (3 points) Prove the following statement using the formal definition of a limit.

$$\lim_{x\to 5} 4x+1 = 21$$

That is, given $\varepsilon > 0$, find a value of $\delta > 0$ so that the formal definition holds.

Solution:

Let $f(x) = 4x+1$. We claim that $\delta = \varepsilon/4$ will work. To see this, we suppose that x satisfies $0 < |x-5| < \varepsilon/4$. Then

$$|f(x)-21| = |4x+1-21| = 4|x-5| < 4 \times \frac{\varepsilon}{4} = \varepsilon$$

3. (4 points) Carefully prove that the equation below has at least one real solution. *Be sure to quote the appropriate theorems.*

$$x^3 + 5x + 1 = 0$$

Solution:

Let $f(x) = x^3 + 5x + 1$. Observe that $f(-2) = -17$ and $f(0) = 1$ and that f is continuous on the interval* $[-2, 0]$ since it is a polynomial. So f satisfies the hypotheses of the Intermediate Value Theorem. It follows that f attains every value between $f(-2)$ and $f(0)$. So there is a point $c \in (-2, 0)$ such that $f(c) = 0$. In other words,

$$c^3 + 5c + 1 = 0$$

which is precisely what we wished to show!

* - Other intervals will work.