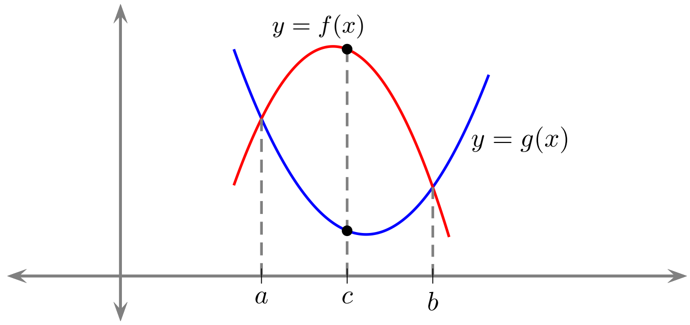
1. Suppose that $0 < a < b$. Prove that there is a point $c \in (a, b)$ such that $c^2 = ab$. (*Hint:* There are at least two ways to proceed. **Method 1:** Let $f(x) = x^2 - ab$ show that $f(x)$ has a zero, etc. **Method 2:** Let $g(x) = 1/x$ and use the MVT.)

2. Let $f(x) = \dfrac{x^3}{3} - x^2 - 3x + 2$. Find the absolute maximum of $f(x)$ on the interval [1, 10] and say where it is attained. **Justify your answer.**

3. (12 points) Solve the following initial value problem.

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}, \qquad y(9) = 0$$

4. Let $f(x)$ and $g(x)$ be differentiable functions that intersect at $a$ and $b$ (see the sketch). Suppose the vertical distance between the curves is **greatest** at $x = c$. Show that the tangent lines at $(c, f(c))$ and $(c, g(c))$ must be parallel.




5. Evaluate the integrals.

   (a) $\displaystyle\int \left(5\sin 2x - 3\sec^2 x\right) dx$

   (b) $\displaystyle\int \left(5x^3 - 2x^2 + \frac{4}{x^2}\right) dx$

   (c) $\displaystyle\int \frac{x}{\sqrt{x+1}}\, dx$ (Hint: $x = x + 1 - 1$.)

6. Let $g(x) = x(x-2)\sqrt{3-x}$. Answer the following questions.

   *Note:* $g'(x) = \dfrac{-5x^2+18x-12}{2\sqrt{3-x}}$ and $g''(x) = \dfrac{-3(x-2)(5x-16)}{4(3-x)^{3/2}}$.

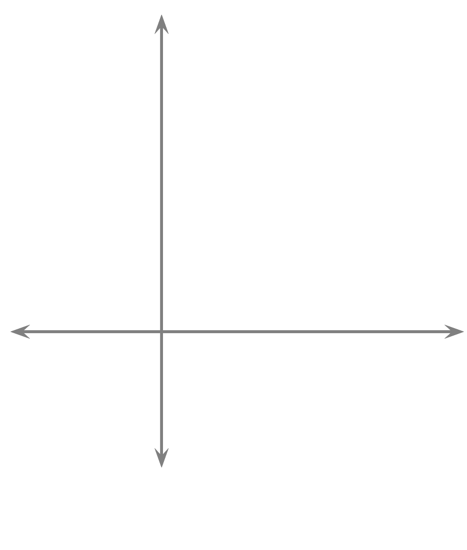
   (a) Identify the intervals on which $g$ is increasing and decreasing. *You may use a monotonicity chart as we have done in class.*

   (b) Identify the intervals on which $g$ is concave up and concave down. *You may use a concavity chart as we have done in class.*

   (c) Identify all local extrema. *Indicate whether the given point is a local maximum or minimum.*

   (d) Identify all inflection points.

   (e) Sketch the graph of $y = g(x)$. Give the coordinates of the local extremes.



7. Let $f$ be continuous on $[0, \infty)$ with $f(0) = 0$. Suppose that $f'(x) \geq 1$ for all $x \in (0, \infty)$. Prove that $f(x) \geq x$ for all $[0, \infty)$.

8. Evaluate the following integrals.

   (a) $\displaystyle\int_3^5 (4x-1)(x+2)\,dx$

   (b) $\displaystyle\int_0^{\pi/3} \cos^2 x \,\sin x\,dx$

   (c) $\displaystyle\int 4t^2\sqrt{2+t^3}\,dt$

9. Suppose that $f'(x) > 0$ for all $x$ and that $f(1) = 0$. Which of the statements below *must* be true about the function

$$g(x) = \int_0^x f(t)\, dt$$

(a) $g$ is a continuous function of $x$.

(b) $g$ is a differentiable function of $x$.

(c) The graph of $y = g(x)$ has a horizontal tangent line at $x = 1$.

(d) $g$ has a local minimum at $x = 1$.

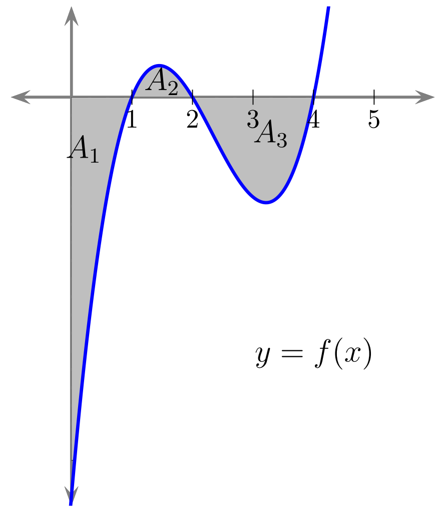
(e) $g$ has a local maximum at $x = 1$.

(f) The graph of $y = g(x)$ has an inflection point at $x = 1$.

(g) The graph of $y = g'(x)$ crosses the $x$-axis at $x = 1$.

(h) The graph of $y = g(x)$ is concave up (everywhere).

10. The graph of $y = f(x)$ is shown below. Answer the following questions. *Note:* The areas of the shaded regions are $A_1 = 3.25$, $A_2 = 0.5$ and $A_3 = 3$.




(a) Find the total area of the shaded region.

(b) Evaluate $\int_1^4 f(x)\,dx$

(c) Evaluate $\int_0^2 3f(x)\,dx - \int_2^4 f(x)\,dx$

(d) What is the average value of $f$ over the interval $[1, 4]$.

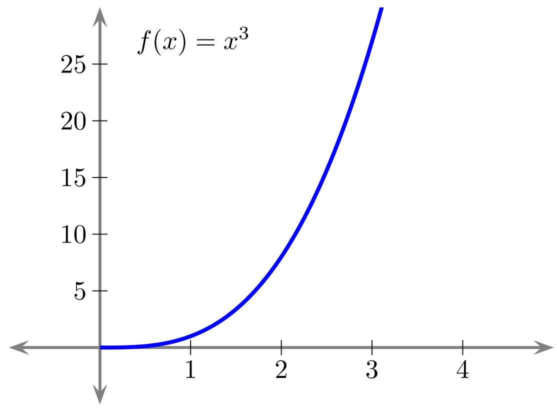
11. Find $dy/dx$ in each of the following.

(a) $y = \int_2^x t \sin t^2\,dt$

(b) $y = \int_0^{\sqrt{x}} (1 + t^2)^6\,dt$

12. *Estimate* the integral below by subdividing the appropriate interval into 4 equal subintervals and evaluating the corresponding Riemann sum using right end points. A SKETCH IS INCLUDED FOR YOUR CONVENIENCE.

$$\int_1^3 x^3\,dx$$




13. In class we observed that $\sin x^2$ has no *elementary* antiderivative. Nevertheless, show that $\int_0^1 \sin x^2\,dx \le 1/3$. (*Hint:* You may freely use the fact that $x^2 - \sin x^2 \ge 0$ for all $x \in \mathbb{R}$.)

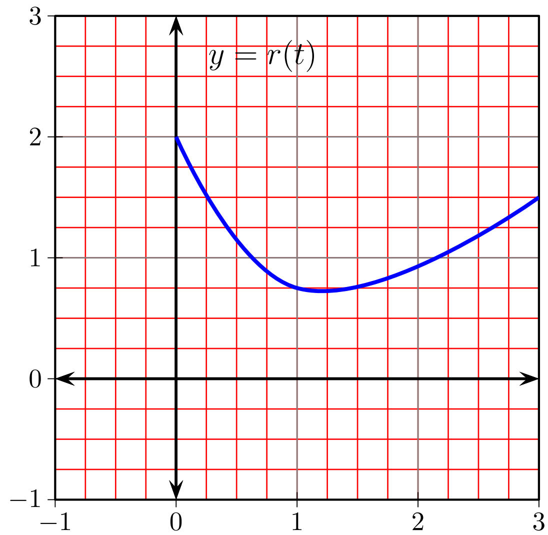
14. The graph of a function $y = r(t)$ shows the rate of change (million bacteria/hour) in the population of a bacteria colony when the colony is treated by a certain drug. Answer the questions below.

(a) What are the units of $\int_0^2 r(t)\,dt$.

(b) What is the practical meaning of $\int_0^2 r(t)\,dt$.

(c) In the sketch, *carefully* shade in the precise region whose area is given by the **right-hand** sum with $n = 4$ (four subdivisions) for the definite integral $\int_0^2 r(t)\,dt$.

(d) Estimate $\int_0^2 r(t)\,dt$ by using the **right-hand** sum with $n = 4$.




15. Use the following facts to answer the questions below. $\int_1^3 f(x)\,dx = 4$, $\int_1^3 g(x)\,dx = -6$, and $\int_1^4 f(x)\,dx = 11$

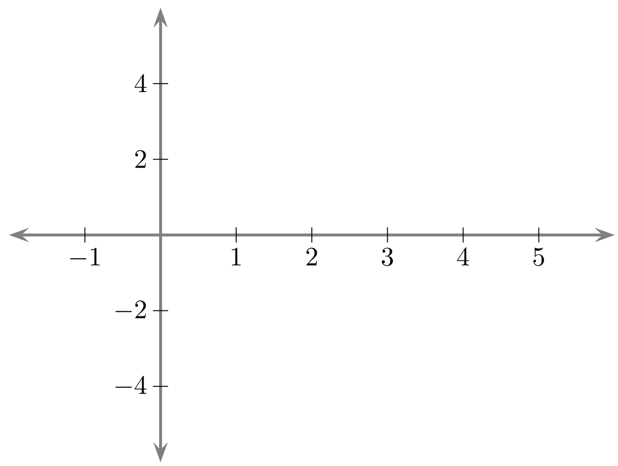
(a) $\int_1^3 (5f(x) - 2g(x))\,dx =$

(b) $\int_3^4 f(x)\,dx =$

(c) If $H(x) = \int_1^x \frac{3^t}{\sqrt{t}}\,dt$, then $H'(x) =$

16. *Carefully* sketch the graph of a **continuous function** which has the following properties.

- $f(1) = 2$
- $f'(1)$ does not exist
- $f'(4) = f''(4) = 0$
- $f'(x) > 0$ for $x < 1$
- $f'(x) < 0$ for $1 < x < 4$ and $x > 4$
- $f''(x) > 0$ for $x < 1$ and $1 < x < 4$
- $f''(x) < 0$ for $x > 4$




17. Evaluate the integrals.

(a) $\displaystyle\int \left( \frac{\cos 3x}{4} + 5 \sec 2x \tan 2x \right) dx$

(b) $\displaystyle\int \left( 5x^4 - 2x^3 - \frac{2}{\sqrt{3x}} \right) dx$

(c) $\displaystyle\int \sin x \cos x \, dx$

18. Prove that $g(x) = 2\sqrt{x}$ is a *contraction mapping* on $[1, \infty)$. That is, show that if $a, b \in [1, \infty)$ then $|g(b) - g(a)| \le |b - a|$. (*Hint:* $x \ge 1 \Longrightarrow \sqrt{x} \ge 1$.)

19. Let $f(x) = \dfrac{x^3}{3} - x^2 - 2x$. Find the absolute **minimum** of $f(x)$ on the interval $[-2, 2]$ and say where it is attained. **Justify your answer.**

20. Find at least one critical point of $p(x) = x\cos^2 x$.

21. Let $g(x) = \dfrac{x^2+3}{x-1}$. Given that $g'(x) = \dfrac{(x+1)(x-3)}{(x-1)^2}$ and $g''(x) = \dfrac{8}{(x-1)^3}$, answer the questions below.

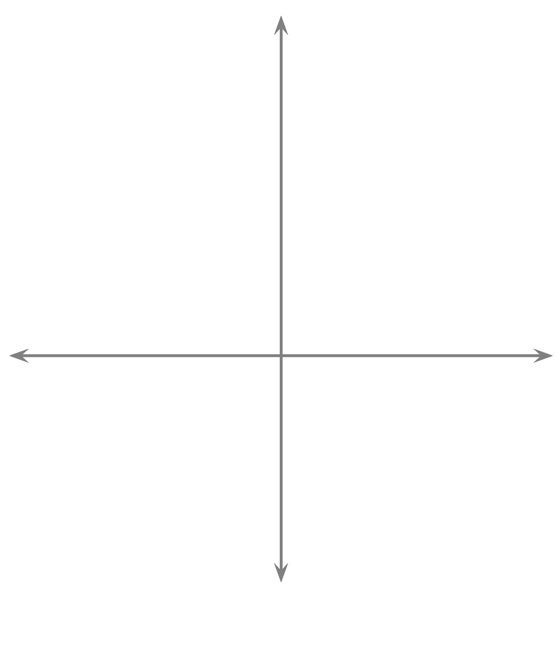
(a) Find the equation(s) of all asymptotes.

(b) Identify the intervals on which $g$ is increasing and decreasing. *You may use a monotonicity chart as we have done in class.*

(c) Identify the intervals on which $g$ is concave up and concave down. *You may use a concavity chart as we have done in class.*

(d) Identify all local extrema. *Indicate whether the given point is a local maximum or minimum.*

(e) Sketch the graph of $y = g(x)$. Give the coordinates of the local extremes. Use dashed lines to indicate the asymptotes.



22. Solve the following initial value problem.

$$\frac{dy}{dx} = \frac{2}{\sqrt{5x}}, \qquad y(5) = 0$$

23. A rectangular metal box is to contain 36 cubic meters and will have a **square** base and top. The material for the base costs $10 per square meter and the material for sides and top costs $6 per square meter. Find the dimensions that minimize cost.

24. Suppose that $g$ is continuous on $[a, b]$ and differentiable on $(a, b)$ and that $g(a) > g(b)$. Show that $g'(x)$ is negative at some $x \in (a, b)$.