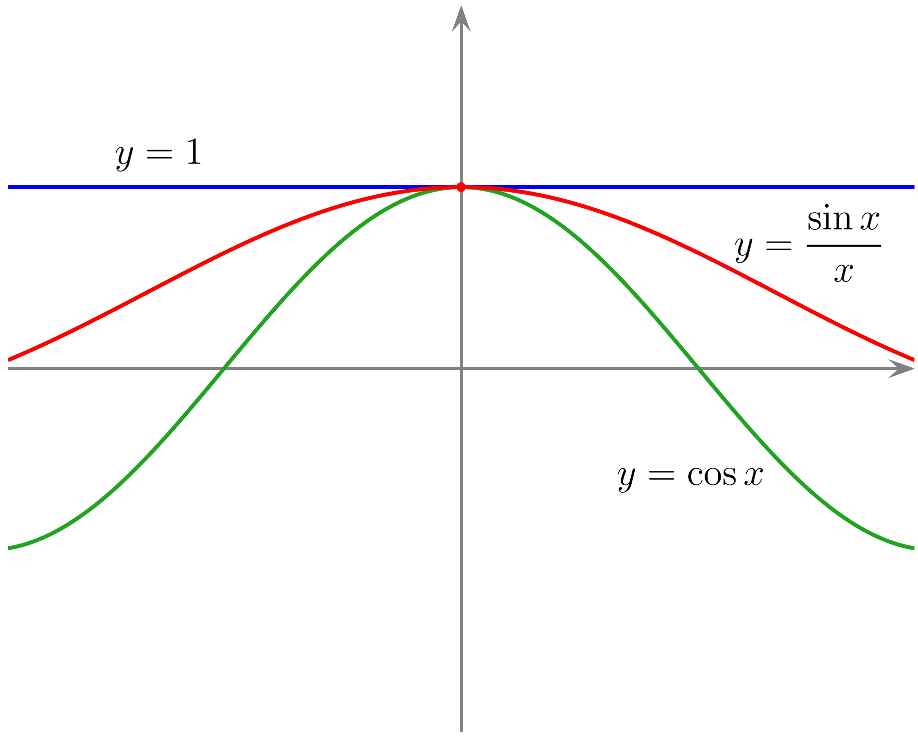


Figure 1: $\cos x < \frac{\sin x}{x} < 1$

Figure 1 illustrates the relationship between the functions $\cos x$, $\frac{\sin x}{x}$, and 1 for values of x near zero. That is, for $x \in A = [-\pi/2, 0) \cup (0, \pi/2]$, one has

$$\cos x < \frac{\sin x}{x} < 1 \tag{1}$$

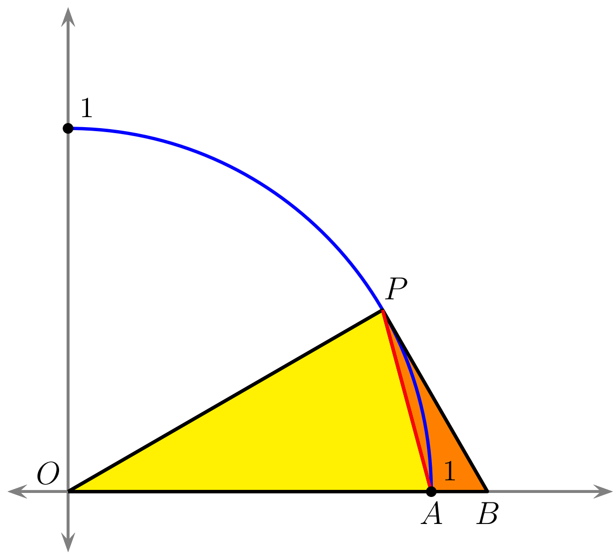
Observe that

$$\lim_{x \to 0} \cos x = 1 = \lim_{x \to 0} 1$$

So if (1) is true, then by the Squeeze Theorem we must have

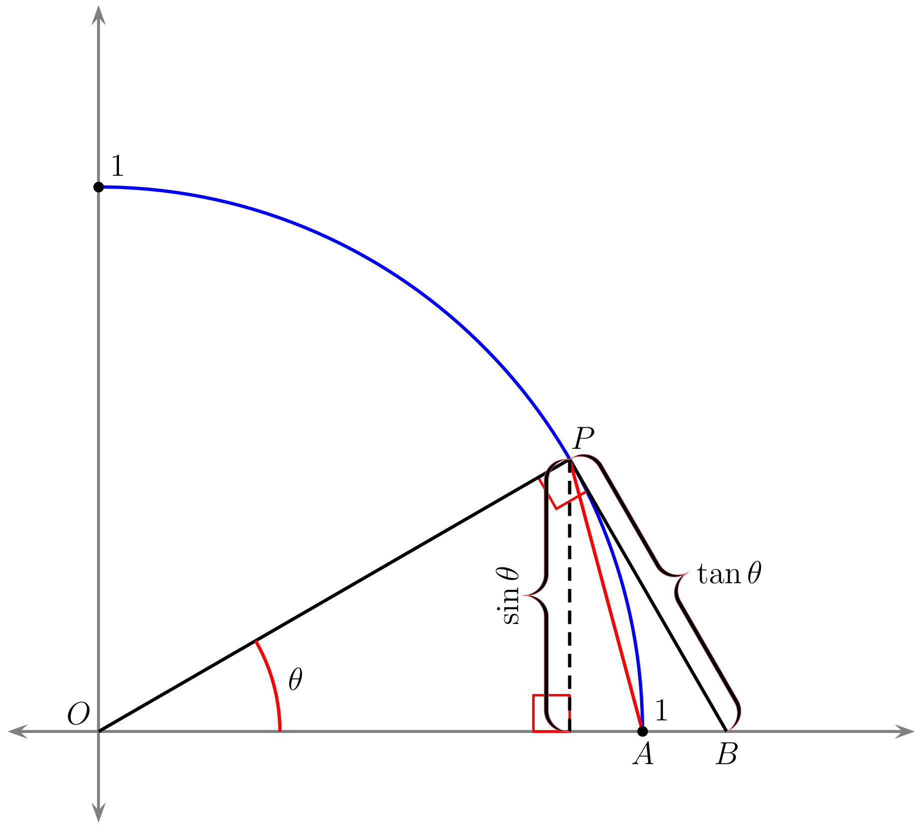
$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

To prove (1), we appeal to a bit of geometry.



Notice that

$$\text{area } \triangle OAP < \text{area of sector } OAP < \text{area } \triangle OBP$$



In other words, if $\pi/2 > \theta > 0$ then

$$\frac{1}{2}\sin\theta < \frac{1}{2}\theta < \frac{1}{2}\tan\theta$$

The first inequality implies that

$$\frac{\sin\theta}{\theta} < 1.$$

The second implies

$$\frac{\tan\theta}{\theta} > 1$$

or

$$\frac{\sin\theta}{\theta} > \cos\theta.$$

Thus

$$\cos\theta < \frac{\sin\theta}{\theta} < 1$$