1. Let $f(x) = \cos\left(\frac{\pi x}{6}\right) + 2x$. Find the average rate of change in the function between $x = -2$ and $x = 4$. *Your answer should be a number.*

Solution:

$$f(4) = \cos\left(\frac{\pi(4)}{6}\right) + 2(4) = -1/2 + 8$$

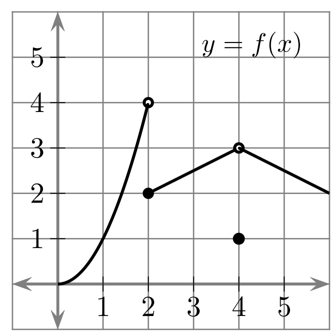
and

$$f(-2) = \cos\left(\frac{\pi(-2)}{6}\right) + 2(-2) = 1/2 - 4$$

It follows that

$$f_{\text{avg}} = \frac{f(4) - f(-2)}{4 - (-2)} = \frac{15/2 - (-7/2)}{6} = 11/6$$

2. Let $y = f(x)$ as shown. Compute the limits that follow (or explain why they do not exist).



(a) $\lim_{x \to 4} f(x) = 3$

(b) $\lim_{x \to 2^-} f(x) = 4$

Solution:

A few comments: Part (a) says that the limit (at 4) exists. Notice however, that the limit at 2 does not exist (why?). In section 1.8 we will learn that f is discontinuous at 2 and 4 (for different reasons), but it is continuous everywhere else. Can the discontinuities be "fixed"? That is, can f be *redefined* at $x = 2$ so that it is continuous there? How about at $x = 4$?

3. Evaluate the limit below.

$$\lim_{x \to -3} \frac{x^2 - 9}{x + 3} =$$

Solution:

$$= \lim_{x \to -3} \frac{(x+3)(x-3)}{x+3} = \lim_{x \to -3} (x - 3) = -6$$

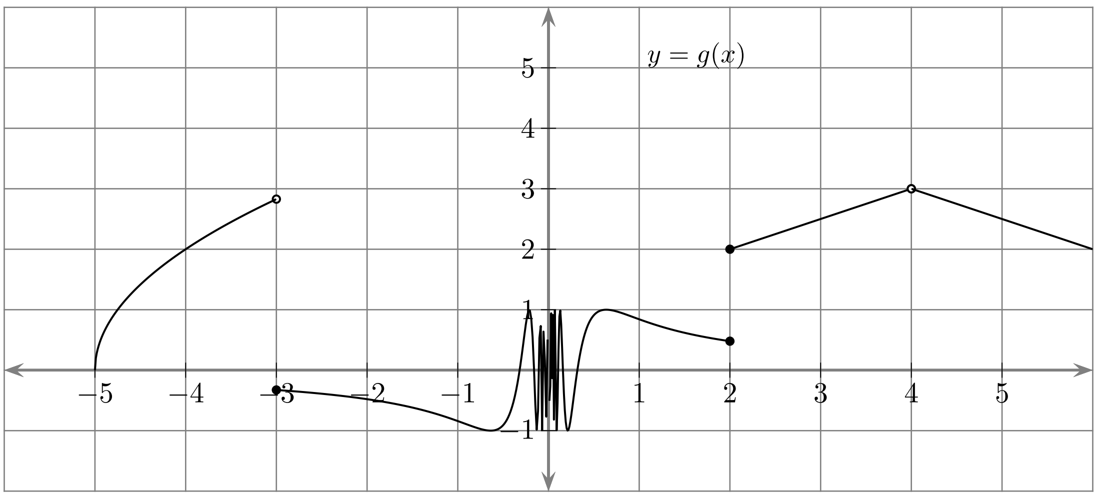


Figure 1: Limits from graphs

4. Use the sketch of $y = g(x)$ shown in Figure 1 to answer the questions below.

   (a) $\lim_{x \to 4} g(x) = 2$

   (b) $\lim_{x \to -3^-} g(x) \approx 2.8$

   (c) $\lim_{x \to 0} g(x) = \text{DNE}$

   (d) Make up your own limit.