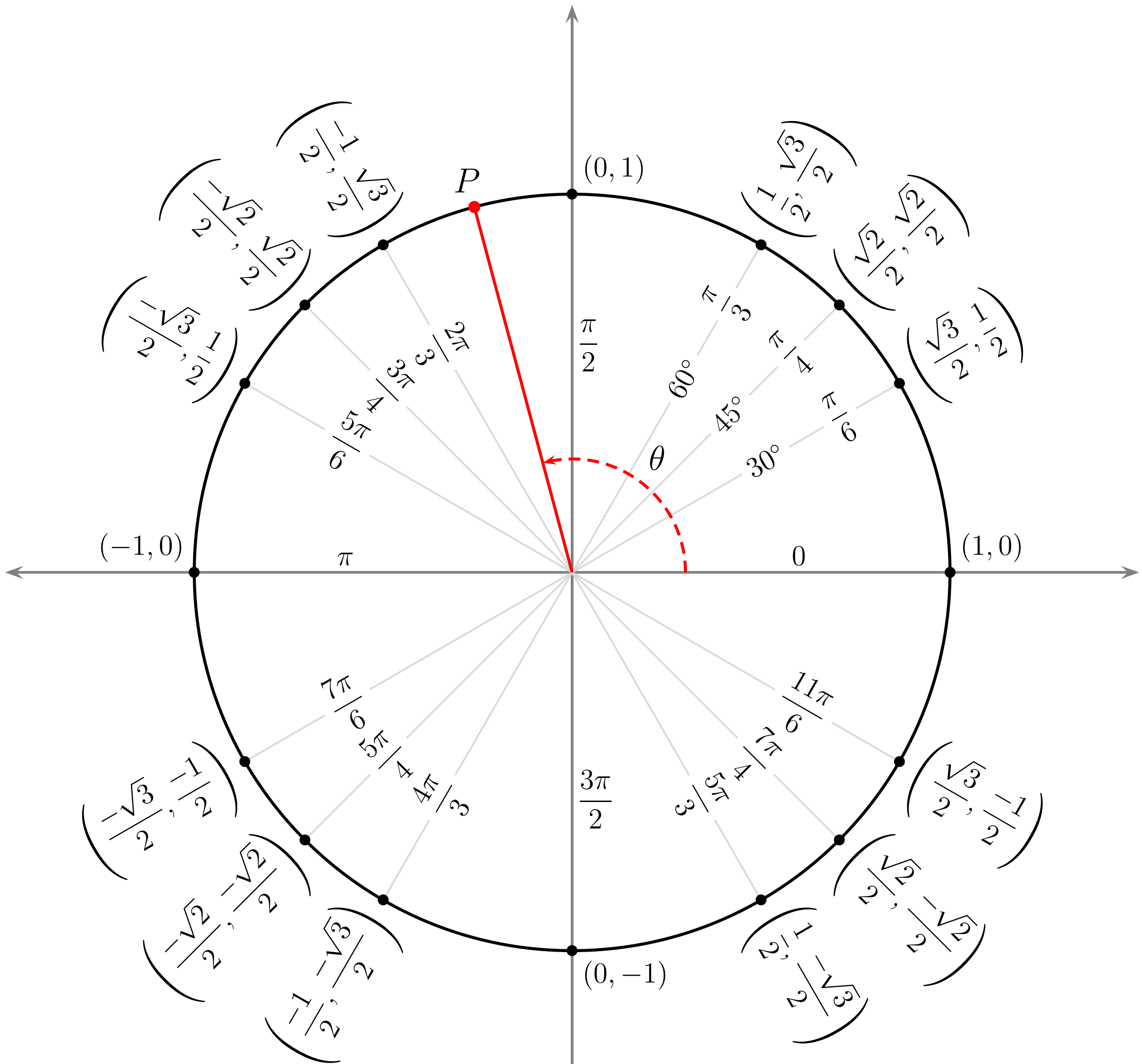

Figure 1: The Unit Circle

Recall: Suppose that $P = P(a, b)$ is a point on the unit circle (with θ given in **standard position**) as shown in Figure 1. Then

$$\sin\theta = b, \quad \cos\theta = a, \quad \tan\theta = b/a, \quad \text{etc.}$$

So, for example,

$$\cos\frac{5\pi}{6} = \frac{-\sqrt{3}}{2} \quad \text{and} \quad \sec\frac{5\pi}{3} = \frac{1}{\cos(5\pi/3)} = \frac{1}{1/2} = 2$$

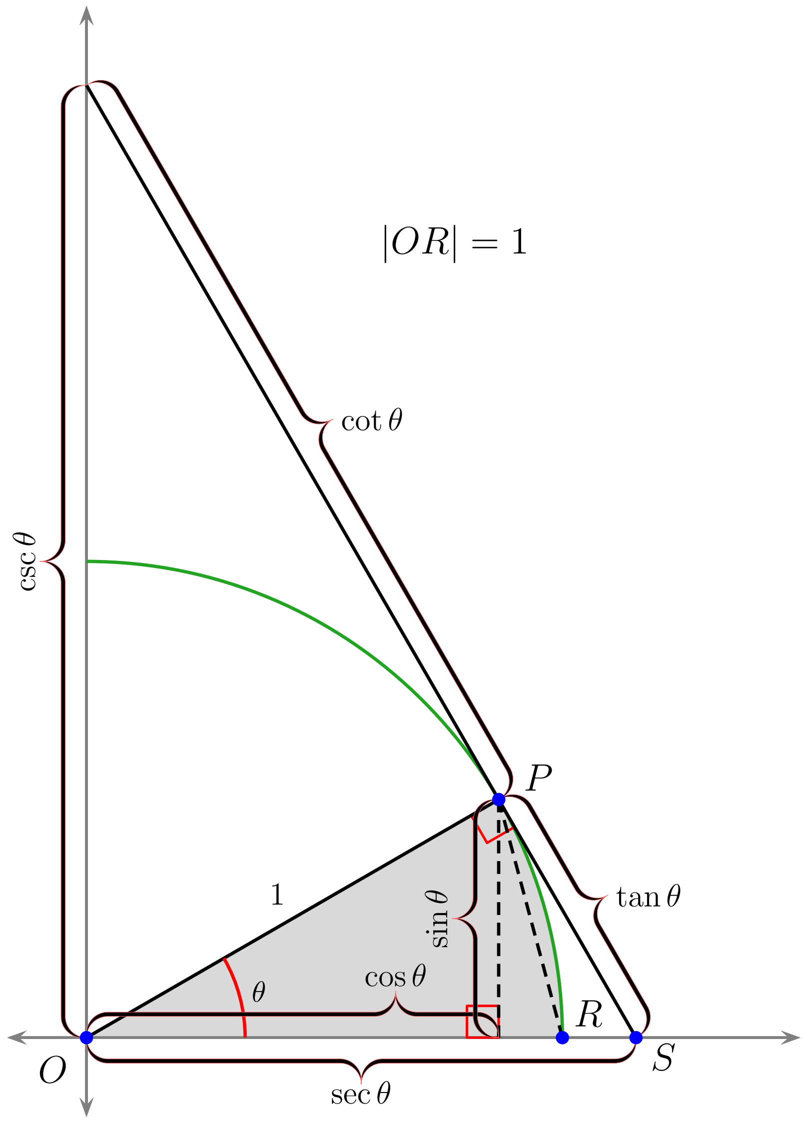

Figure 2: The green arc is part of the unit circle.

From Figure 2 we see that the area of the shaded region, **sector** OPR, is bounded by the areas of the triangles, ΔOPR and $\triangle OPS$. Specifically,

$$a(\triangle OPR) < a(\textbf{sector}OPR) < a(\triangle OPS)$$

Since the formula for the area of a sector with central angle θ (measured in radians) with radius r is $\theta r^2/2$, we obtain the important inequality

$$\frac{1}{2}(1)\sin\theta < \frac{1}{2}\theta(1)^2 < \frac{1}{2}(1)\tan\theta \qquad \text{or}$$
$$\sin\theta < \theta < \tan\theta \tag{1}$$

To see why this is important, notice that the sine function is positive for $0 < \theta < \pi/2$. Then the left-hand inequality yields

$$0 < \sin\theta < \theta$$

From this we see that

$$\lim_{\theta\to 0^+} \sin\theta = 0$$

We will revisit inequality (1) in some later sections.