Review for Final Exam.

- Exam is cumulative.
- Heat equation not included.
- 15 problems.
- Two and half hours.
- Fourier Series expansions (Chptr.6).
- Eigenvalue-Eigenfunction BVP (Chptr. 6).
- Systems of linear Equations (Chptr. 5).
- Laplace transforms (Chptr. 4).
- Second order linear equations (Chptr. 2).
- ▶ First order differential equations (Chptr. 1).

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Graph the odd-periodic extension of f(x) = 1 for $x \in (-1, 0)$, and then find the Fourier Series of this extension.

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Solution: Recall:
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We conclude:
$$f(x) = -\frac{4}{\pi} \sum_{k=1}^{\infty} \frac{1}{(2k-1)} \sin[(2k-1)\pi x].$$

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$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

Example

Graph the odd-periodic extension of f(x) = 2 - x for $x \in (0, 2)$, and then find the Fourier Series of this extension.

Solution: The Fourier series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right].$$

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$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx = \frac{2}{L} \int_{0}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx,$$

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Since f is odd and periodic, then the Fourier Series is a Sine Series, that is, $a_n = 0$.

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$$b_n = \int_0^2 (2-x) \sin\left(\frac{n\pi x}{2}\right) dx.a$$

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Solution:
$$b_n = 2 \int_0^2 \sin\left(\frac{n\pi x}{2}\right) dx - \int_0^2 x \sin\left(\frac{n\pi x}{2}\right) dx.$$
$$\int \sin\left(\frac{n\pi x}{2}\right) dx = \frac{-2}{n\pi} \cos\left(\frac{n\pi x}{2}\right),$$

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Example

Graph the odd-periodic extension of f(x) = 2 - x for $x \in (0, 2)$, and then find the Fourier Series of this extension.

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Solution:
$$b_n = 2 \int_0^2 \sin\left(\frac{n\pi x}{2}\right) dx - \int_0^2 x \sin\left(\frac{n\pi x}{2}\right) dx.$$
$$\int \sin\left(\frac{n\pi x}{2}\right) dx = \frac{-2}{n\pi} \cos\left(\frac{n\pi x}{2}\right),$$

The other integral is done by parts,

$$I=\int x\sin\Bigl(\frac{n\pi x}{2}\Bigr)\,dx,$$

Example

Graph the odd-periodic extension of f(x) = 2 - x for $x \in (0, 2)$, and then find the Fourier Series of this extension.

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So, we get
$$b_n = 2\left[\frac{-2}{n\pi} \cos\left(\frac{n\pi x}{2}\right)\right]\Big|_0^2 + \left[\frac{2x}{n\pi} \cos\left(\frac{n\pi x}{2}\right)\right]\Big|_0^2 - \left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi x}{2}\right)\Big|_0^2$$

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$$b_n = \frac{-4}{n\pi} \left[\cos(n\pi) - 1\right] + \left[\frac{4}{n\pi} \cos(n\pi) - 0\right]$$

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$$b_n = \frac{-4}{n\pi} [\cos(n\pi) - 1] + \left[\frac{4}{n\pi} \cos(n\pi) - 0\right] \Rightarrow b_n = \frac{4}{n\pi}.$$

Example

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 $b_n = 2\left[\frac{-2}{n\pi} \cos\left(\frac{n\pi x}{2}\right)\right]\Big|_0^2 + \left[\frac{2x}{n\pi} \cos\left(\frac{n\pi x}{2}\right)\right]\Big|_0^2 - \left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi x}{2}\right)\Big|_0^2$
 $b_n = \frac{-4}{n\pi} \left[\cos(n\pi) - 1\right] + \left[\frac{4}{n\pi} \cos(n\pi) - 0\right] \Rightarrow b_n = \frac{4}{n\pi}.$
We conclude: $f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{2}\right).$

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Fourier Series: Even/Odd-periodic extensions.

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$$A_{(2k-1)} = \frac{1}{(2k-1)^2 \pi^2} \begin{bmatrix} 1 - (-1) \end{bmatrix} = \frac{1}{(2k-1)^2 \pi^2}$$

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Review for Final Exam.

- Fourier Series expansions (Chptr.6).
- ► Eigenvalue-Eigenfunction BVP (Chptr. 6).
- Systems of linear Equations (Chptr. 5).
- Laplace transforms (Chptr. 4).
- Second order linear equations (Chptr. 2).
- First order differential equations (Chptr. 1).

Example

Find the positive eigenvalues and their eigenfunctions of

$$y'' + \lambda y = 0$$
, $y(0) = 0$, $y(8) = 0$.

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Solution: Since $\lambda > 0$, introduce $\lambda = \mu^2$, with $\mu > 0$.

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$$\lambda = \left[\frac{(2n+1)\pi}{16}\right]^2, \quad y_n(x) = \sin\left(\frac{(2n+1)\pi x}{16}\right). \qquad \triangleleft$$

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Example

Find the non-negative eigenvalues and their eigenfunctions of

$$y'' + \lambda y = 0$$
, $y'(0) = 0$, $y'(8) = 0$.

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$$c_1 = -\frac{\sqrt{3/2}}{1/2}$$

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$$c_1 = -\frac{\sqrt{3}/2}{1/2} = -\sqrt{3}$$

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Example

Find the solution of the BVP

$$y'' + y = 0$$
, $y'(0) = 1$, $y(\pi/3) = 0$.

Solution: $y(x) = e^{rx}$ implies that r is solution of $p(r) = r^2 + \mu^2 = 0 \implies r_{\pm} = \pm i.$

The general solution is $y(x) = c_1 \cos(x) + c_2 \sin(x)$.

Then, $y'(x) = -c_1 \sin(x) + c_2 \cos(x)$. The B.C. imply:

$$1 = y'(0) = c_2 \quad \Rightarrow \quad y(x) = c_1 \cos(x) + \sin(x).$$

$$0 = y(\pi/3) = c_1 \cos(\pi/3) + \sin(\pi/3) \implies c_1 = -\frac{\sin(\pi/3)}{\cos(\pi/3)}.$$

$$c_1 = -\frac{\sqrt{3}/2}{1/2} = -\sqrt{3} \Rightarrow y(x) = -\sqrt{3}\cos(x) + \sin(x).$$

Review for Final Exam.

- Fourier Series expansions (Chptr.6).
- Eigenvalue-Eigenfunction BVP (Chptr. 6).
- Systems of linear Equations (Chptr. 5).
- Laplace transforms (Chptr. 4).
- Second order linear equations (Chptr. 2).
- First order differential equations (Chptr. 1).

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$$\mathbf{x}^{(1)} = \mathbf{v} \, e^{\lambda t}, \quad \mathbf{x}^{(2)} = (\mathbf{v} \, t + \mathbf{w}) \, e^{\lambda t}.$$

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Summary: Find solutions of x' = Ax, with A a 2 × 2 matrix.
First find the eigenvalues λ_i and the eigenvectors v⁽ⁱ⁾ of A.
(c) If λ₁ = λ₂ = λ, real, and their eigenvectors {v⁽¹⁾, v⁽²⁾} are linearly independent, then the general solution is

 $\mathbf{x}(x) = c_1 \, \mathbf{v}^{(1)} \, e^{\lambda t} + c_2 \, \mathbf{v}^{(2)} \, e^{\lambda t}.$

(d) If $\lambda_1 = \lambda_2 = \lambda$, real, and there is only one eigendirection **v**, then find **w** solution of $(A - \lambda I)\mathbf{w} = \mathbf{v}$. Then fundamental solutions to the differential equation are given by

$$\mathbf{x}^{(1)} = \mathbf{v} \, e^{\lambda t}, \quad \mathbf{x}^{(2)} = (\mathbf{v} \, t + \mathbf{w}) \, e^{\lambda t}.$$

Then, the general solution is

$$\mathbf{x} = c_1 \, \mathbf{v} \, e^{\lambda t} + c_2 \left(\mathbf{v} \, t + \mathbf{w}
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Example

Find the solution to:
$$\mathbf{x}' = A\mathbf{x}$$
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A - 3I

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A + 3I

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$$\begin{bmatrix} c_1\\c_2 \end{bmatrix} = \frac{1}{(2+1)} \begin{bmatrix} 1 & 1\\-1 & 2 \end{bmatrix} \begin{bmatrix} 3\\2 \end{bmatrix} \implies \begin{bmatrix} c_1\\c_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 5\\1 \end{bmatrix} .$$

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$$\begin{bmatrix} c_1\\c_2 \end{bmatrix} = \frac{1}{(2+1)} \begin{bmatrix} 1 & 1\\-1 & 2 \end{bmatrix} \begin{bmatrix} 3\\2 \end{bmatrix} \implies \begin{bmatrix} c_1\\c_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 5\\1 \end{bmatrix}.$$
We conclude: $\mathbf{x}(t) = \frac{5}{3} \begin{bmatrix} 2\\1 \end{bmatrix} e^{3t} + \frac{1}{3} \begin{bmatrix} -1\\1 \end{bmatrix} e^{-3t}.$

Review for Final Exam.

- Fourier Series expansions (Chptr.6).
- Eigenvalue-Eigenfunction BVP (Chptr. 6).
- Systems of linear Equations (Chptr. 5).
- ► Laplace transforms (Chptr. 4).
- Second order linear equations (Chptr. 2).
- First order differential equations (Chptr. 1).

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Summary:

► Main Properties:

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 $\mathcal{L}[f^{(n)}(t)] = s^n \mathcal{L}[f(t)] - s^{(n-1)} f(0) - \dots - f^{(n-1)}(0); \quad (18)$

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 $\mathcal{L}[(f * g)(t)] = \mathcal{L}[f(t)]\mathcal{L}[g(t)].$

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Partial fraction decompositions, completing the squares.

Example

Use L.T. to find the solution to the IVP

$$y'' + 9y = u_5(t),$$
 $y(0) = 3,$ $y'(0) = 2.$

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Solution: Compute $\mathcal{L}[y''] + 9 \mathcal{L}[y] = \mathcal{L}[u_5(t)]$

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Solution: Compute $\mathcal{L}[y''] + 9\mathcal{L}[y] = \mathcal{L}[u_5(t)] = \frac{e^{-5s}}{s}$,

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$$\mathcal{L}[y''] = s^2 \, \mathcal{L}[y] - s \, y(0) - y'(0)$$

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$$\mathcal{L}[y''] = s^2 \mathcal{L}[y] - s y(0) - y'(0) \quad \Rightarrow \quad \mathcal{L}[y''] = s^2 \mathcal{L}[y] - 3s - 2.$$

$$(s^2+9)\mathcal{L}[y]-3s-2=\frac{e^{-5s}}{s}$$

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$$(s^{2}+9)\mathcal{L}[y] - 3s - 2 = \frac{e^{-3s}}{s}$$
$$\mathcal{L}[y] = \frac{(3s+2)}{(s^{2}+9)} + e^{-5s} \frac{1}{s(s^{2}+9)}$$

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$$H(s) = \frac{1}{s(s^2+9)}$$

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$$1 = as^2 + 9a + bs^2 + cs$$

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Partial fractions on

$$H(s) = \frac{1}{s(s^2+9)} = \frac{a}{s} + \frac{(bs+c)}{(s^2+9)} = \frac{a(s^2+9) + (bs+c)s}{s(s^2+9)},$$
$$1 = as^2 + 9a + bs^2 + cs = (a+b)s^2 + cs + 9a$$

Example

Use L.T. to find the solution to the $\ensuremath{\mathsf{IVP}}$

$$y'' + 9y = u_5(t),$$
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Therefore, we conclude that,

$$y(t) = 3\cos(3t) + \frac{2}{3}\sin(3t) + \frac{u_5(t)}{9} \Big[1 - \cos(3(t-5)) \Big].$$

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Review for Final Exam.

- Fourier Series expansions (Chptr. 6).
- Eigenvalue-Eigenfunction BVP (Chptr. 6).
- Systems of linear Equations (Chptr. 5).
- Laplace transforms (Chptr. 4).
- ► Second order linear equations (Chptr. 2).

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First order differential equations (Chptr. 1).

Summary: Solve $y'' + a_1 y' + a_0 y = g(t)$.

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$$y'' - 2y' - 3y = 3e^{-t}, \quad y(0) = 1, \quad y'(0) = \frac{1}{4}.$$

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$$y'(t) = 3c_1 e^{3t} - c_2 e^{-t} - \frac{3}{4}(e^{-t} - t e^{-t}).$$

$$1 = y(0) = c_1 + c_2,$$

Example

Find the solution y to the initial value problem

$$y'' - 2y' - 3y = 3e^{-t}, \quad y(0) = 1, \quad y'(0) = \frac{1}{4}$$

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$$c_1 + c_2 = 1, \\ 3_1 - c_2 = 1 \end{cases} \Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Example

S

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Since $c_1 = \frac{1}{2}$ and $c_2 = \frac{1}{2}$,

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Since $c_1 = \frac{1}{2}$ and $c_2 = \frac{1}{2}$, we obtain,
 $y(t) = \frac{1}{2} (e^{3t} + e^{-t}) - \frac{3}{4}t e^{-t}.$

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Review for Final Exam.

- Fourier Series expansions (Chptr.6).
- Eigenvalue-Eigenfunction BVP (Chptr. 6).
- Systems of linear Equations (Chptr. 5).
- Laplace transforms (Chptr. 4).
- Second order linear equations (Chptr. 2).
- ▶ First order differential equations (Chptr. 1).

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Summary:

• Linear, first order equations: y' + p(t)y = q(t).

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• Linear, first order equations: y' + p(t)y = q(t).

Use the integrating factor method: $\mu(t) = e^{\int p(t) dt}$.

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Summary:

Linear, first order equations: y' + p(t) y = q(t).
 Use the integrating factor method: μ(t) = e^{∫ p(t) dt}.

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• Separable, non-linear equations: h(y) y' = g(t).

Summary:

- Linear, first order equations: y' + p(t) y = q(t).
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$$\int h(u)\,du=\int g(t)\,dt+c.$$

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Summary:

- Linear, first order equations: y' + p(t) y = q(t).
 Use the integrating factor method: μ(t) = e^{∫ p(t) dt}.
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The solution can be found in implicit of explicit form.

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► No modeling problems from Sect. 2.3.

Summary:

▶ Bernoulli equations: $y' + p(t) y = q(t) y^n$, with $n \in \mathbb{R}$.

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N(x,y)y'+M(x,y)=0.

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If the equation is exact, then there is a potential function ψ , such that $N = \partial_y \psi$ and $M = \partial_x \psi$.

The solution of the differential equation is

 $\psi(x,y(x))=c.$

Advice: In order to find out what type of equation is the one you have to solve, check from simple types to the more difficult types:

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 Exact equation with integrating factor. (Very complicated to check.)

Example

Find all solutions of $y' = \frac{x^2 + xy + y^2}{xy}$.

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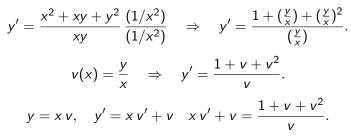
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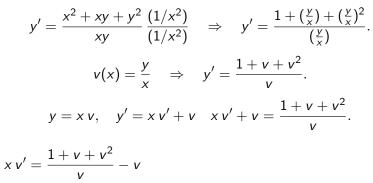
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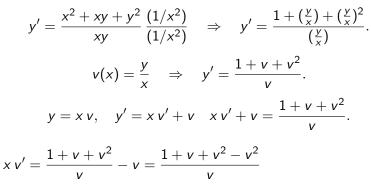
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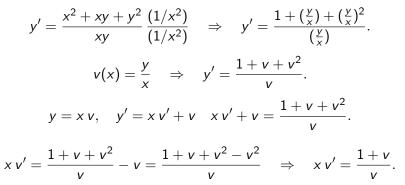
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Use the substitution u = 1 + v, hence du = v'(x) dx.

Example Find all solutions of $y' = \frac{x^2 + xy + y^2}{xy}$. Solution: Recall: $v' = \frac{1+v}{v}$. This is a separable equation. $\frac{v(x)}{1+v(x)}v'(x) = \frac{1}{x} \Rightarrow \int \frac{v(x)}{1+v(x)}v'(x) dx = \int \frac{dx}{x} + c$.

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Example

Find the solution y to the initial value problem

$$y' + y + e^{2x} y^3 = 0, \qquad y(0) = \frac{1}{3}.$$

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Use the integrating factor method.

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$$e^{-2x} v' - 2 e^{-2x} v = 2 \quad \Rightarrow \quad \left(e^{-2x} v\right)' = 2.$$

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$$y' + y + e^{2x} y^3 = 0,$$
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The initial condition y(0) = 1/3

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$$[2x^{2}y + 2x] y' + [2xy^{2} + 2y] = 0.$$

$$N = [2x^{2}y + 2x] \Rightarrow \partial_{x}N = 4xy + 2.$$

$$M = [2xy^{2} + 2y] \Rightarrow \partial_{y}M = 4xy + 2.$$

$$\Rightarrow \partial_{x}N = \partial_{y}M.$$

The equation is exact. There exists a potential function ψ with

$$\partial_y \psi = N, \qquad \partial_x \psi = M.$$

$$\partial_y \psi = 2x^2y + 2x \quad \Rightarrow \quad \psi(x, y) = x^2y^2 + 2xy + g(x).$$

$$2xy^2 + 2y + g'(x) = \partial_x \psi = M = 2xy^2 + 2y \quad \Rightarrow \quad g'(x) = 0.$$

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$$\psi(x, y) = x^2y^2 + 2xy + c, \qquad x^2y^2(x) + 2xy(x) + c = 0. \quad \triangleleft$$