

Hw 21 - Prblm. 2

Solve the system:

$$x_1 - x_2 + 3x_3 = -2 \quad (1)$$

$$3x_1 - 4x_2 + 6x_3 = -9 \quad (2)$$

$$3x_1 + 18x_3 = 3 \quad (3)$$

Substitution:

From eq. (1): $x_1 = -2 + x_2 - 3x_3$ Substitute in (2), (3).
(1)

First in eq. (2)

$$3(-2 + x_2 - 3x_3) - 4x_2 + 6x_3 = -9$$

$$-6 + 3x_2 - 9x_3 - 4x_2 + 6x_3 = -9$$

$$-x_2 - 3x_3 = -3 \quad \Rightarrow \quad x_2 + 3x_3 = 3 \quad (2)$$

Now in eq. (3)

$$3(-2 + x_2 - 3x_3) + 18x_3 = 3$$

$$-6 + 3x_2 - 9x_3 + 18x_3 = 3$$

$$3x_2 + 9x_3 = 9 \quad \Rightarrow \quad x_2 + 3x_3 = 3 \quad (3)$$

Since (2) is equal to (3), we only have one eq. for two unknowns, x_2, x_3 .

(2) \Rightarrow $x_2 = 3 - 3x_3$ Substitute x_2 in (1)

$$x_1 = -2 + (3 - 3x_3) - 3x_3$$

$$x_1 = 1 - 6x_3 \quad \text{and} \quad x_3 : \text{free.}$$

The solution $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is then given by

$$\underline{x} = \begin{bmatrix} 1 - 6x_3 \\ 3 - 3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -6x_3 \\ -3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -6 \\ -3 \\ 1 \end{bmatrix} x_3$$

$$\boxed{\underline{x} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -6 \\ -3 \\ 1 \end{bmatrix} x_3}, \quad x_3 \in \mathbb{R}.$$