1. (20 points) Use the Laplace transform to find the solution \( y \) to the initial value problem

\[ y'' + 3y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 3. \]
2. (20 points) Use the Laplace transform to find the solution $y$ to the initial value problem

$$y'' + 2y = -2\delta(t - 3), \quad y(0) = 0, \quad y'(0) = 0.$$
3. (15 points) Use convolutions to express the function $f$ whose Laplace transform is

$$\mathcal{L}[f(t)] = \frac{1}{(s^2 + 3)(s^2 - 4)}.$$
4. (a) (20 points) Find the general solution $\mathbf{x}$ to the $2 \times 2$ linear system

$$\mathbf{x}'(t) = A \mathbf{x}(t), \quad A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}.$$ 

(b) (5 points) Sketch a qualitative phase portrait of the solution trajectories.
5. (20 points) Find the solution $x$ to the initial value problem

$$x'(t) = Ax(t), \quad x(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}.$$