1. (30 points) Verify that the functions $y_1(t) = t$ and $y_2(t) = te^t$ are solutions to the homogeneous differential equation

$$t^2y'' - t(t + 2)y' + (t + 2)y = 0 \quad t > 0,$$

and then use the method of variation of parameters to obtain a particular solution to the inhomogeneous differential equation

$$t^2y'' - t(t + 2)y' + (t + 2)y = t^3e^{3t} \quad t > 0.$$

2. (35 points) Decide whether the set of vectors shown below is linearly dependent or independent. In the case that the set of vectors is linearly dependent, express one of them as a linear combination of the other two.

$$\begin{align*}
\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 7 \\ 4 \end{bmatrix} \right\}
\end{align*}$$

3. (35 points) Find all eigenvalues and eigenvectors of matrix $A$ below. Also find all eigenvalues and eigenvectors of the matrix $B$ below,

$$A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$