1. (20 points) Find the solution $y(x)$ to the initial value problem

$$2y'' + 8y' + 10y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$
2. (20 points) Use the method of undetermined coefficients to find the general solution $y(x)$ of the inhomogeneous equation

$$y'' + 4y = 3\sin(2x).$$
3. (20 points) Use the method of variation of parameters to find a particular solution $y(x)$ to the inhomogeneous equation

$$(1 - x) y'' + x y' - y = 2(1 - x)^2 e^x, \quad x \neq 1,$$

knowing that the functions $y_1(x) = e^x$ and $y_2(x) = x$ are solutions of the homogeneous equation

$$(1 - x) y'' + x y' - y = 0.$$
4. (20 points) Find the recurrence relation for the coefficients $a_n$ of the power series expansion $y(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n$ centered at $x_0 = 3$ of the solution of the differential equation

$$2 y'' + (x + 1) y' + 3 y = 0.$$ 

You do not need to solve the recurrence relation.
5. (20 points) Find the general solution \( y(x) \) of the Euler equation

\[ x^2 y'' + 3x y' + 5y = 0. \]