1. (25 points) Find the solution $y(t)$ to the initial value problem

$$t y' + 2 y = \sin(t), \quad t > 0, \quad y(2\pi) = \frac{1}{2\pi}.$$
2. (a) (20 points) Compute an *implicit* expression for the solution \( y(x) \) to the initial value problem

\[
y' = \frac{x(x^2 + e^x)}{4y^3}, \quad y(0) = -\sqrt{2}.
\]

(b) (5 points) Find the *explicit* expression for the solution found in part (2a).
3. (25 points) A tank contains a volume $V_0 = 100$ gallons of water with a $Q_0$ amount of salt dissolved in it. At a time $t_0 = 0$ minutes fresh water is pouring into the tank at a constant rate $r_i$, while water is also leaving the tank at a constant rate $r_o$ with a salt concentration $q_o(t)$. Consider that there is a mixing mechanism in the tank such that the salt that enters into the tank is *instantaneously mixed* in the tank.

Find the values of the rates $r_i$ and $r_o$ such that the following two conditions hold: First, the volume of water in the tank remains constant; second, the time needed to reduce the initial amount of salt $Q_0$ in the tank to the value $e^{-5}Q_0$ is precisely 25 minutes.
4. (25 points) Show that the following differential equation is exact and then find an implicit expression for all solutions \( y(x) \). The differential equation is the following:

\[
[x^2 + y^2] [x + y y'(x)] + 2 = 0.
\]