1. (25 points) Find the solution $y(t)$ to the initial value problem

$$ty' + 2y = \sin(t), \quad t > 0, \quad y(2\pi) = \frac{1}{2\pi}.$$
2. (a) (20 points) Compute an implicit expression for the solution $y(x)$ to the initial value problem

$$y' = \frac{x(x^2 + e^x)}{4y^3}, \quad y(0) = -\sqrt{2}.$$ 

(b) (5 points) Find the explicit expression for the solution found in part (2a).
3. (25 points) A tank contains a volume $V_0 = 100$ gallons of water with a $Q_0$ amount of salt dissolved in it. At a time $t_0 = 0$ minutes fresh water is pouring into the tank at a constant rate $r_i$, while water is also leaving the tank at a constant rate $r_o$ with a salt concentration $q_o(t)$. Consider that there is a mixing mechanism in the tank such that the salt that enters into the tank is *instantaneously mixed* in the tank.

Find the values of the rates $r_i$ and $r_o$ such that the following two conditions hold: First, the volume of water in the tank remains constant; second, the time needed to reduce the initial amount of salt $Q_0$ in the tank to the value $e^{-5}Q_0$ is precisely 25 minutes.
4. (25 points) Show that the following differential equation is exact and then find an implicit expression for all solutions $y(x)$. The differential equation is the following:

$$\left[ x^2 + y^2 \right] \left[ x + y y'(x) \right] + 2 = 0.$$