Print Name: ______________________ **Section Number:** ______

TA Name: ______________________ **Section Time:** ______

Math 20C
Final Exam.
June 15, 2006

No calculators or any other devices are allowed on this exam.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
Read each question carefully. If any question is not clear, ask for clarification.
Answer each question completely, and show all your work.

1. (10 points) Find the plane through the point $P_0 = (2, -1, 1)$ which is perpendicular to the planes $2x - y - z = 3$ and $x + 2y + z = 2$.

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2. (8 points) Decide whether the $\lim_{(x,y)\to(0,0)} \dfrac{y^4 - x^2}{y^4 + x^2}$ exists. Give reasons your answer.

3. (8 points) Does the function $f(x, y, z) = 2z^3 - 3(x^2 + y^2)z$ satisfy the Laplace equation $f_{xx} + f_{yy} + f_{zz} = 0$? Give reasons your answer.

4. (10 points) Find the linear approximation $L(x, y)$ of the function $f(x, y) = \sqrt{11 - x^2 - y^2}$ at the point $(1, 1)$. Use this approximation to estimate the value of $f(0.9, 1.2)$.

5. (10 points) Find the local maxima, local minima and saddle points of the function $f(x, y) = x^3 + y^3 - 3x^2 + 3y^2 - 8$.

6. (10 points) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = \dfrac{1}{x} - \dfrac{1}{y}$ subject to the constraint $\dfrac{1}{x^2} + \dfrac{1}{y^2} = 1$.

7. Consider the integral $\int\int_D f(x,y)\,dA = \int_0^2 \int_{-3\sqrt{1-\frac{x^2}{2^2}}}^{3(1-\frac{x}{2})} f(x,y)\,dy\,dx$.

(a) (8 points) Sketch the region of integration.

(b) (8 points) Switch the order of integration in the above integral.

(c) (8 points) Compute the integral $\int\int_D f(x,y)\,dA$ for the case $f(x,y) = xy$.

8. (10 points) Transform to polar coordinates and then evaluate the integral

$$I = \int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \left(x^2 + y^2\right)^{3/2} \, dy \, dx.$$

9. (10 points) Find the volume of a parallelepiped whose base is a rectangle in the $z = 0$ plane given by $0 \leqslant y \leqslant 2$ and $0 \leqslant x \leqslant 1$, while the top side lies in the plane $x+y+z = 3$.