

Print Name: _____ Student Number: _____

Section Time: _____

Math 20C.
Final Exam
December 8, 2005

Read each question carefully, and answer each question completely.
Show all of your work. No credit will be given for unsupported answers.
Write your solutions clearly and legibly. No credit will be given for illegible solutions.

1. (8 Pts.) Find the equation of the plane that contains both the point $(1, 0, -1)$ and the line $x = t, y = -1 + 2t, z = 3t$.

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2. (8 Pts.) Find the values of the constants b and c such that the function

$$f(t, x) = \sin(x - bt) + \cos(cx + t)$$

is solution of the wave equation $f_{tt} = 9f_{xx}$.

3. (6 Pts.) Consider the function $z(t) = f(x(t), y(t))$, where

$$f(x, y) = (x^2 + 2y)^{1/2}, \quad x(t) = e^{3t}, \quad y(t) = e^{-3t}.$$

Compute $\frac{d}{dt}z(t)$.

4. (8 Pts.) The function $z(x, y)$ is defined implicitly by the equation $z^2xy = \sin(2y + z)$. Compute the partial derivatives $\partial z/\partial x$ and $\partial z/\partial y$ as functions of x , y and z .

5. (10 Pts.) Reparametrize the curve $\mathbf{r}(t) = (e^{3t} \cos(3t), 1, e^{3t} \sin(3t))$, with respect to the arc length measured from the point where $t = 0$ in the direction of increasing t .

6. (10 Pts.) Consider the function $f(x, y) = x^3 - 6xy + 8y^3$.

(a) Find the critical points of f .

(b) For each critical point determine whether it is a local maximum, local minimum, or a saddle point.

7. (10 Pts.) Find the maximum and minimum values of the function $f(x, y) = 2x^2 + y^2 - 2y$ subject to the constraint $x^2 + y^2 = 4$.

8. (a) (10 Pts.) Sketch the region of integration, D , whose area is given by the double integral

$$\int \int_D dA = \int_0^3 \int_{\frac{2}{3}x}^{2\sqrt{x/3}} dy dx.$$

- (b) Compute the double integral given in (a).
(c) Change the order of integration in the integral given in (a). (You don't need to compute the integral again.)

9. (10 Pts.) Compute the integral

$$I = \int_0^1 \int_0^x \int_0^{4-x^2} xz \, dy \, dz \, dx$$

10. (10 Pts.) Consider the region of $D \subset \mathbb{R}^3$ given by

$$D = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq 1, \quad 0 \leq z \leq 1 + x^2 + y^2\}.$$

- (a) Sketch the region D .
- (b) Compute the volume of that region.