Name: _____ **Section:** _____

TA: _____ **Time:** _____

Math 21C.
Final Examination
June 10, 2003

Read each question carefully, and answer each question completely.
Show all of your work. No credit will be given for unsupported answers.
Write your solutions clearly and legibly. No credit will be given for illegible solutions.

1. Find an equation for the plane that passes through the point $(1, 2, 3)$ and contains the line $x = 3t$, $y = 1 + t$, $z = 2 - t$.

#	Score
1	
2	
3	
4	
5	
6	
7	
8	
Σ	

2. The position of a particle at time t is given by $\mathbf{r}(t) = \langle 2\cos t, 3t, 2\sin t \rangle$.

   (a) Find the velocity at time t.

   (b) Find the acceleration at time t.

   (c) Find the angle between the velocity and acceleration at time t.

3. Find the linear approximation $L(x, y)$ to $f(x, y) = \ln(x - 3y)$ at the point $(7, 2)$.

4. The gradient of a function $f(x, y, z)$ at the point $(3, 4, -5)$ is $\langle 1, -2, 2 \rangle$.

   (a) Find the values of the partial derivatives f_x, f_y, and f_z at the point $(3, 4, -5)$.

   (b) Find the maximum rate of change of f at the point $(3, 4, -5)$ and the unit vector in the direction which the maximum rate of change occurs.

5. Find the point on the plane $2x - y + z = 1$ that is closest to the point $(-4, 1, 3)$.

6. Consider the iterated integral $\int_{y=0}^{\sqrt{\pi}} \int_{x=y}^{\sqrt{\pi}} \sin(x^2)\, dx\, dy$.

(a) Sketch the region integration; clearly label each part of its boundary with the appropriate equation.

(b) Evaluate the integral. Reverse the order of integration, if necessary.

7. Evaluate $\int_{y=0}^{1} \int_{x=0}^{\sqrt{1-y^2}} \cos(x^2+y^2)\, dx\, dy$. You may change to polar coordinates if you wish.

8. Let T be the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $2x+y+z = 2$. Compute $\iiint_T x^2 \, dV$.