Name: ____________________________ Section Number: ______

TA Name: ____________________________ Section Time: ______

Math 20C
Midterm Exam 2.
May 26, 2006

No calculators or any other devices are allowed on this exam.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
Read each question carefully. If any question is not clear, ask for clarification.
Answer each question completely, and show all your work.

1. (a) (5 points) Find and sketch the domain of the function \( f(x, t) = \ln(2x + 3t) \).

   (b) (5 points) Find all possible constants \( c \) such that the function \( f(x, t) \) above is solution of the wave equation, \( f_{tt} - c^2 f_{xx} = 0 \).
2. (a) (5 points) Find the direction in which \( f(x, y) \) increases the most rapidly, and the directions in which \( f(x, y) \) decreases the most rapidly at \( P_0 \), and also find the value of the directional derivative of \( f(x, y) \) at \( P_0 \) along these directions, where
\[
f(x, y) = x^2 e^{3y}, \quad \text{and} \quad P_0 = (-1, 0).
\]

(b) (5 points) Find the directional derivative of \( f(x, y) \) above at the point \( P_0 \) in the direction given by \( \mathbf{v} = (-1, 1) \).
3. (a) (5 points) Find the tangent plane approximation of \( f(x, y) = x \cos(\pi y/2) - y^2 e^x \) at the point \((0, -1)\).

(b) (5 points) Use the linear approximation computed above to approximate the value of \( f(0.1, -0.9) \).
4. (10 points) Find every local and absolute extrema of \( f(x, y) = y^2 + 3x^2 + 2x \) on the unit disk \( x^2 + y^2 \leq 1 \), and indicate which ones are the absolute extrema. In the case of the interior stationary points, decide whether they are local maximum, minimum of saddle points.