1. (4 points) Consider the vectors $\vec{v} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{w} = \hat{i} + 2\hat{j} - \hat{k}$.

(a) Compute $\vec{v} \cdot \vec{w}$.

$$\vec{v} \cdot \vec{w} = \langle 2, -2, 1 \rangle \cdot \langle 1, 2, -1 \rangle = 2 - 4 - 1 = -3.$$ 

(b) What is the cosine of the angle between $\vec{v}$ and $\vec{w}$?

$$|\vec{v}| = \sqrt{4+4+1} = 3, \quad |\vec{w}| = \sqrt{1+4+1} = \sqrt{6}.$$ 

$$\cos(\theta) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{-3}{3\sqrt{6}} = -\frac{1}{\sqrt{6}}.$$ 

(c) Find a unit vector in the direction of $\vec{v}$.

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{3} \langle 2, -2, 1 \rangle.$$ 

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2. (4 points) Find the equation of the plane that contains the lines \( \mathbf{r}_1(t) = \langle 1, 2, 3 \rangle t \) and \( \mathbf{r}_2(t) = \langle 1, 1, 0 \rangle + \langle 1, 2, 3 \rangle t \).

\( P_0 = (1, 1, 0) \) is in the plane. \( P_1 = (1, 2, 3) = \mathbf{r}_1(t = 1) \) is also in the plane.

Therefore, \( P_0 P_1 = (0, 1, 3) \) is tangent to the plane.

\( \mathbf{v} = \langle 1, 2, 3 \rangle \) is also tangent to the plane. Then, the normal vector to the plane \( \mathbf{n} \) can be computed as follows:

\[
\mathbf{n} = \mathbf{v} \times P_0 P_1 = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & 2 & 3 \\
0 & 1 & 3 \\
\end{vmatrix} = (6 - 3)\mathbf{i} - (3 - 0)\mathbf{j} + (1 - 0)\mathbf{k} = \langle 3, -3, 1 \rangle.
\]

Then, the equation of the plane can be constructed with \( P_0 = (1, 1, 0) \) and \( \mathbf{n} = \langle 3, -3, 1 \rangle \) as follows:

\[
3(x - 1) - 3(y - 1) + z = 0,
\]

\[
3x - 3y + z = 0.
\]
3. (4 points) Find an equation for the plane that passes through the points (2, 2, 0), (1, 0, 3), and (0, 1, 2).

Call the points \( P = (2, 2, 0) \), \( Q = (1, 0, 3) \), and \( R = (0, 1, 2) \).

Introduce the vectors \( P\vec{R} = (-2, -1, 2) \), and \( P\vec{Q} = (-1, -2, 3) \).

They are tangent to the plane.

So the normal vector to the plane \( \vec{n} \) can be computed as follows:

\[
\vec{n} = P\vec{R} \times P\vec{Q} = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
-2 & -1 & 2 \\
-1 & -2 & 3 \\
\end{vmatrix} = (-3 - (-4))\vec{i} - (-6 - (-2))\vec{j} + (4 - 1)\vec{k} = (1, 4, 3).
\]

Then, the equation of the plane passing through \( P = (2, 2, 0) \) with normal \( \vec{n} = (1, 4, 3) \) is

\[
(x - 2) + 4(y - 2) + 3(z - 0) = 0,
\]

\[
x + 4y + 3z = 10.
\]
4. (4 points) A particle moves along the curve \( \mathbf{r}(t) = (\sin(3t), 4t, \cos(3t)) \), for \( t \geq 0 \).

(a) Find the velocity \( \mathbf{v}(t) \) and acceleration \( \mathbf{a}(t) \) functions of the particle.

\[
\mathbf{v}(t) = (3 \cos(3t), 4, -3 \sin(3t)), \\
\mathbf{a}(t) = (-9 \sin(3t), 0, -9 \cos(3t)).
\]

(b) Reparametrize the curve \( \mathbf{r}(t) \) with respect to the arc length measured from the point where \( t = 0 \), in the direction of increasing \( t \).

\[
\ell(t) = \int_0^t |\mathbf{v}(u)| \, du, \\
= \int_0^t \sqrt{9 \cos^2(3u) + 16 + 9 \sin^2(3u)} \, du, \\
= \int_0^t \sqrt{9 + 16} \, du, \\
= 5t.
\]

Then,
\[
t = \frac{\ell}{5}.
\]

The reparametrized curve \( \mathbf{r}(\ell) \) is then given by
\[
\mathbf{r}(\ell) = \left( \sin \left( \frac{3}{5} \ell \right), \frac{4}{5} \ell, \cos \left( \frac{3}{5} \ell \right) \right).
\]