1. (6 points) Consider the vectors \( \mathbf{v} = (6, 2, -3) \) and \( \mathbf{w} = (-2, 2, 1) \).

(a) Find a vector normal to both, \( \mathbf{v} \) and \( \mathbf{w} \).

\[
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
6 & 2 & -3 \\
-2 & 2 & 1 \\
\end{vmatrix}
= (2 + 6)\mathbf{i} - (6 - 6)\mathbf{j} + (12 + 4)\mathbf{k}
= (8, 0, 16).
\]

A solution is any vector proportional to \( \mathbf{v} \times \mathbf{w} \). For example \( \mathbf{u} = (1, 0, 2) \).

(b) Find the area of the parallelogram formed by \( \mathbf{v} \) and \( \mathbf{w} \).

\[
A = |\mathbf{v} \times \mathbf{w}| = \sqrt{8^2 + 16^2} = \sqrt{8^2(1 + 4)} = 8\sqrt{5}.
\]

(c) Find a vector of length one in the direction of \( \mathbf{w} \).

\[
|\mathbf{w}| = \sqrt{4 + 4 + 1} = \sqrt{9} = 3.
\]

\[
\mathbf{u} = \frac{\mathbf{w}}{|\mathbf{w}|} = \frac{1}{3}(-2, 2, 1).
\]
2. (6 points) Find an equation for the plane that passes through the points $(-1, 1, 1)$, $(-1, -1, 1)$, and $(0, 0, 2)$.

Let

$$P = (-1, 1, 1), \quad Q = (-1, -1, 1), \quad R = (0, 0, 2).$$

Then,

$$\vec{PQ} = (0, -2, 0), \quad \vec{PR} = (1, -1, 1),$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -2 & 0 \\ 1 & -1 & 1 \end{vmatrix} = (-2 - 0)\mathbf{i} - (0 - 0)\mathbf{j} + (0 + 2)\mathbf{k} = (-2, 0, 2).$$

Take $\mathbf{n} = (-2, 0, 2)$, and a point $R = (0, 0, 2)$. Then, the equation of the plane is

$$-2(x - 0) + 0(y - 0) + 2(z - 2) = 0,$$

$$-x + z = 2.$$
3. (6 points) Consider the line given by \(\mathbf{r}(t) = \langle 0, 1, 1 \rangle + \langle 1, 2, 3 \rangle t\) and the plane given by \(2x + y - z = -1\).

(a) Does the line intersect the plane? If yes, find the intersection point. In any case, justify your answer.

The line
\[
x = t, \quad y = 1 + 2t, \quad z = 1 + 3t,
\]
intersect the plane \(2x + y - z = -1\) if there is a solution \(t\) for the equation
\[
2t + (1 + 2t) - (1 + 3t) = -1, \quad \Rightarrow t = -1.
\]
Therefore, the point of intersection has coordinates \(x = -1, y = -1, z = -2\), then
\[
P = (-1, -1, -2).
\]

(b) Find the equation of the line, passing through the point \((0, -1, -1)\) and orthogonal to the plane given above.

The tangent to the line is the normal to the plane, so the result is
\[
\mathbf{r}(t) = \langle 0, 1, 1 \rangle + \langle 2, 1, -1 \rangle t.
\]
4. (6 points) A particle moves in a plane with a velocity function given by the expression 
\[ v(t) = (3 \sin(t), 2 \cos(t)), \text{ for } t \geq 0. \]

(a) Find the acceleration \( a(t) \) function of the particle.

\[ a(t) = v'(t) = (3 \cos(t), -2 \sin(t)). \]

(b) Find the position function \( r(t) \) of the particle knowing that the initial position of 
the particle is \( r(0) = (-1, 1) \).

\[ r(t) = (-3 \cos(t) + r_{0x}, 2 \sin(t) + r_{0y}). \]
\[ r(0) = (-3 + r_{0x}, r_{0y}) = (-1, 1), \Rightarrow r_{0x} = 2, \quad r_{0y} = 1. \]

Then, the position function is

\[ r(t) = (-3 \cos(t) + 2, 2 \sin(t) + 1). \]