Instructions: Fifty-five minutes. No books or notes; graphing calculators without symbolic manipulation programs are permitted. Do all 6 problems in your blue book. Show all work; unsubstantiated answers will not receive credit. Turn in your exam sheet with your blue book.

1. (20 points) The function \( g(x) \) defined on the interval \((0, 3)\) satisfies \( g(1) = 3 \) and \( g'(1) = -1 \).
   
   (a) Find an equation for the line tangent to the graph \( y = g(x) \) at \( x = 1 \).
   
   (b) Find the value of \( \lim_{x \to 1} \frac{g(x) - 3}{x - 1} \).

2. (20 points) Find an exact value of \( \lim_{x \to 0} \frac{\sqrt{3} - x - \sqrt{3}}{x} \) and justify your answer.

3. (20 points) A car drives down a road and is at distance (in miles) \( d(t) = 60 \left( \frac{1}{1 + t} - \frac{1}{7} \right) \) from its destination after \( t \) hours.
   
   (a) What is the average velocity while traveling between \( t = 2 \) and \( t = 3 \)?
   
   (b) Express its instantaneous velocity at time \( t = 2 \) as a limit.
   
   (c) Compute the limit.

   Be sure to indicate the units in (a), (b), and (c) above.

4. (20 points) Prove that there is at least one negative real number \( x \) satisfying the equation \( x^3 - x + 2 = 0 \). (You must use a theorem, not just a graph.)

5. (20 points) Find all the horizontal asymptote(s) of the curve \( y = \frac{x + 3}{\sqrt{9x^2 + 3}} \). Justify your answer.

6. (20 points) Let \( a > 0 \) and consider

\[
   f(x) = \begin{cases} 
   5 - ax^2, & x < 1; \\
   ax^2 + 2, & x \geq 1. 
   \end{cases}
\]

Show that there is a unique value of \( a \) such that \( f \) is continuous at every real number. (A correct value of \( a \) will not be sufficient; you must justify your answer.)