Math 20A
First Midterm Exam. October 22, 2002
VERSION 2

Instructions: *Fifty-five minutes. No books or notes; graphing calculators without symbolic manipulation programs are permitted. Do all 6 problems in your blue book. Show all work; unsubstantiated answers will not receive credit. Turn in your exam sheet with your blue book.*

1. (20 points) The function $g(x)$ defined on the interval $(0, 3)$ satisfies $g(1) = 3$ and $g'(1) = -1$.

   (a) Find an equation for the line tangent to the graph $y = g(x)$ at $x = 1$.

   (b) Find the value of $\lim_{x\to 1} \dfrac{g(x) - 3}{x - 1}$.

2. (20 points) Find an exact value of $\lim_{x\to 0} \dfrac{\sqrt{3-x} - \sqrt{3}}{x}$ and justify your answer.

3. (20 points) A car drives down a road and is at distance (in miles) $d(t) = 60\left(\dfrac{1}{1+t} - \dfrac{1}{7}\right)$ from its destination after t hours.

   (a) What is the average velocity while traveling between $t = 2$ and $t = 3$?

   (b) Express its instantaneous velocity at time $t = 2$ as a limit.

   (c) Compute the limit.

   Be sure to indicate the units in (a), (b), and (c) above.

4. (20 points) Prove that there is at least one negative real number x satisfying the equation
   $x^3 - x + 2 = 0$. (You must use a theorem, not just a graph.)

5. (20 points) Find all the horizontal asymptote(s) of the curve $y = \dfrac{x+3}{\sqrt{9x^2+3}}$. Justify your answer.

6. (20 points) Let $a > 0$ and consider

$$f(x) = \begin{cases} 5 - ax^2, & x < 1; \\ a^x + 2, & x \geq 1. \end{cases}$$

   Show that there is a unique value of a such that f is continuous at every real number. (A correct value of a will not be sufficient; you must justify your answer.)