Math 20A
Second Midterm Exam. November 19, 2002
VERSION 1

Instructions: Fifty-five minutes. No books or notes; graphing calculators without symbolic
manipulation programs are permitted. Do all 5 problems in your blue book. Show all work;
unsubstantiated answers will not receive credit. Turn in your exam sheet with your blue
book.

1. (20 points) Differentiate the following functions:
   (a) $x^3 \ln x$.
   (b) $\sin (e^{7x^2})$.

2. (20 points) Use differentials to estimate the volume of paint required to cover the
   surface of a cube, with sides of length 10 inches, with a 0.025 inch thick coat of paint.

3. (20 points) If a sphere of ice melts so that its surface area decreases at a rate of 1
   cm$^2$/min, find the rate at which the diameter decreases when the diameter is 20 cm.
   (Recall that the surface area of a sphere of radius $r$ is $4\pi r^2$.)

4. (20 points) Let $f$ be a continuous function on $[1, 5]$, differentiable on $(1, 5)$, with $f(1) = 0$ and $f'(x) \geq 1$ for $1 < x < 5$. Find the smallest possible value of $f(5)$. Justify your
   answer.

5. (40 points) Let $f(x) = 3xe^{-x}$.
   (a) Find the local maxima and local minima of $f$, if any.
   (b) Find the intervals on which $f$ is increasing and the intervals on which $f$ is de-
       creasing.
   (c) Find the inflection points of the graph of $f$.
   (d) Find the intervals on which the graph of $f$ is concave up and the intervals on
       which the graph of $f$ is concave down.