Math 20A
Second Midterm Exam. November 19, 2002
VERSION 1

Instructions: *Fifty-five minutes. No books or notes; graphing calculators without symbolic manipulation programs are permitted. Do all 5 problems in your blue book. Show all work; unsubstantiated answers will not receive credit. Turn in your exam sheet with your blue book.*

1. (20 points) Differentiate the following functions:
   (a) $x^3 \ln x$.
   (b) $\sin\left(e^{7x^2}\right)$.
2. (20 points) Use differentials to estimate the volume of paint required to cover the surface of a cube, with sides of length 10 inches, with a 0.025 inch thick coat of paint.
3. (20 points) If a sphere of ice melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 20 cm. (Recall that the surface area of a sphere of radius r is $4\pi r^2$.)
4. (20 points) Let f be a continuous function on $[1,5]$, differentiable on $(1,5)$, with $f(1) = 0$ and $f'(x) \geq 1$ for $1 < x < 5$. Find the smallest possible value of $f(5)$. Justify your answer.
5. (40 points) Let $f(x) = 3xe^{-x}$.
   (a) Find the local maxima and local minima of f, if any.
   (b) Find the intervals on which f is increasing and the intervals on which f is decreasing.
   (c) Find the inflection points of the graph of f.
   (d) Find the intervals on which the graph of f is concave up and the intervals on which the graph of f is concave down.