Math 20A
Final Exam. December 9, 2002
VERSION 1

Instructions: *No books or notes; graphing calculators without symbolic manipulation programs are permitted. Do all 10 problems in your blue book. Show all work; unsubstantiated answers will not receive credit. Turn in your exam sheet with your blue book.*

1. (20 points) Find the following derivatives and justify your answers:

   (a) $\dfrac{d}{dx}\displaystyle\int_1^x \ln t \, dt.$

   (b) $\dfrac{d}{dx}\displaystyle\int_0^{x^2} e^{t^2} \, dt.$

2. (20 points) (20 points) Evaluate the following integrals:

   (a) $\displaystyle\int_0^2 \frac{dx}{x+5}.$

   (b) $\displaystyle\int_0^{26} \frac{dx}{(1+x)^{1/3}}.$

3. (20 points) Evaluate the following integrals:

   (a) $\displaystyle\int_{-2}^{2} (2-|x|) \, dx.$

   (b) $\displaystyle\int_{-2}^{6} (2-|x|) \, dx.$

4. (20 points) Show that $e^{-1} \leq \displaystyle\int_0^1 e^{-x^2} \, dx \leq 1.$

5. (20 points) Find the following limits and justify your answers:

   (a) $\displaystyle\lim_{x\to 0} \frac{1-\cos(2x)}{x^2}.$

   (b) $\displaystyle\lim_{x\to 0} x^2 \ln x.$

6. (20 points) A particle is moving along the curve $y = \sqrt{1+x^2}$. As the particle passes through the point $P = (1, \sqrt{2})$, its x-coordinate increases at a rate of 2 cm/s. How fast is the distance to the origin changing at this time?

7. (20 points)A particle is moving on a straight line with an acceleration of $a(t) = t + \cos t$. Find the position s of the particle as a function of time t if its velocity and position at $t = 0$ are $v(0) = -3$ and $s(0) = 1$, respectively.

8. (20 points) 10 m^2 of material is available to make a rectangular closed box whose height is 1 m. Find the largest possible volume of the box.

9. (20 points) Consider the function defined for all real numbers given by

$$f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(a) Show that f is continuous at 0.

(b) Using the definition of derivative, show that f is differentiable at 0 and evaluate $f'(0)$.

10. (20 points) A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. how should the wire be cut so that the total area enclosed is

(a) maximum?

(b) minimum?