Research interests – Gabriel Nagy

My research interests have been in the area of partial differential equations that arise in mathematical physics, and I have been mainly concerned with existence and uniqueness of solutions to such equations. The Einstein field equations, which describe gravitational interactions between massive objects in space, have occupied a central role in my research. I have also studied the well-posedness of different formulations of Maxwell's equations for electromagnetic fields and the classical field formulation of the Yang-Mills equations, mainly as model problems for the Einstein equations.

In the past few years, my research efforts have concerned the initial and the initial-boundary value problems for Einstein's equations. Interest in these problems within the relativity community has recently increased with the construction of gravitational wave detectors, such as LIGO in the United States, VIGRO and GEO in Europe, and TAMA in Japan[1]. The general theory of relativity predicts that gravitational effects can propagate as waves in the space-time. The direct measure of gravitational waves will open a new window in observational astronomy, providing valuable information about neutron stars, nuclear matter and even black holes, which are regions of space-time where gravity is so strong that nothing can escape. The analysis of these measurements requires comparison with the results of numerical simulations of the observed sources. It is in this step, solving Einstein's equations using a computer, where the initial-boundary value problem for Einstein's equations must be understood. The initial-boundary value problem for Einstein's equations is more complicated than for a standard wave equation on a curved background, due to the presence of gauge freedom and nonlinear constraint equations. Results from my work in those areas can be found in [4, 5, 12, 13].

In the past year, I have been also concerned with the problem of finding initial data for Einstein's equations. A standard approach in finding such initial data is to solve a coupled nonlinear elliptic system, known as the Lichnerowicz-York conformal rescaled system, and then to use that solution to construct the initial data fields for Einstein's equations. I have focused on two main aspects of this problem: First, to find weak solutions to the Lichnerowicz-York equations representing low differentiability initial data for Einstein's equations; second, to find solutions of the Lichnerowicz-York equations representing general initial data for Einstein's equations, where "general" means that there is no smallness restriction on the space derivatives of the mean extrinsic curvature of the resulting initial data. Results on this project can be found in [6, 8, 9]. A side project that I have been working on in the past year is the study of bifurcation phenomena in a family of static solutions of equations that generalize Einstein's equations to a high number of space dimensions. My main interest in this project was to understand basic predictions of these alternative theories of gravitation as well as to learn essential ideas in bifurcation theory. Preliminary results on this project can be found in [7].

The main focus of my research in the coming year will move somewhat towards the understanding of initial and initial-boundary value problems whose solutions describe black holes. I have already started a project to understand the main results available in the literature, which refer to initial data describing black holes [3, 11]. In these articles is proven the existence of a particular type of solution to the Lichnerowicz-York equations which provide initial data for Einstein's equations containing a black hole. These results are obtained assuming several restrictions on the equation coefficients. I plan to understand and, if possible, generalize such results. In the rest of the year I will focus on finding an initial-boundary value formulation for Einstein's equations suitable for the description of space-times containing one or more black holes. Since the physical fields become singular as we approach the black hole world-line, and the characteristic curves of the evolution equations change from being time-like to being null and space-like, this type of initial-boundary value problem is not yet well-understood. Another interesting feature of this initial-boundary value problem is that it will likely be formulated using a mixed elliptic-hyperbolic system of equations. The elliptic equations will describe the main properties of the underlying coordinate system where the initial-boundary value problem is formulated, while the hyperbolic part will describe the behavior of the physical

[1]More information on these gravitational wave detectors can be obtained at the websites `www.ligo.caltech.edu`, `www.virgo.infn.it`, `geo600.aei.mpg.de`, and `tamago.mtk.nao.ac.jp`.

properties of the system. Such elliptic-hyperbolic systems will also be studied as part of a three-year NSF award DMS/CM 0715146, with M. Holst, D. Estep and myself being the PIs. We plan to study finite element approximations of solutions to such systems and to establish stability of consistent approximation schemes, which will imply convergence of the approximate solution to the analytical solution.

My research plan for the following few years is not as well delineated. A subject I have always been attracted to is to show global in time existence of solutions to the Einstein field equations. This problem seems to be extremely difficult to solve, although progress has been made in the last several years [1, 10]. This problem is also tightly related to what might be the greatest unsolved problem in general relativity: the cosmic censorship hypothesis. The main idea behind this hypothesis is the assertion that black holes in the physical world are covered by a region in space-time that avoids any type of information exchange between the black hole and the exterior, where observers live. I have planned to consider this problem in the simplified situation of axially symmetric space-times, which in this context would describe an axially symmetric rotating black hole. I think that the initial-boundary value problem for elliptic-hyperbolic systems I will study in the coming year will play a central role in this research.

Finally, there will be many side projects that could arise in the following years. The understanding and generalizations of gluing methods for solutions of the Einstein constraint equations begun in [2] seems to be a promising field of research. An interesting question is the following: Given a solution of the Einstein constraint equations in a ball on Euclidean space, is it possible to extend this solution smoothly to an asymptotically Euclidean solution of the constraints in the whole Euclidean space? An affirmative answer to this question would imply that gravitational effects have the non-intuitive property of shielding the source of these effects from the external viewer. Such a property would be similar to the shielding in the external electromagnetic field of the charge configurations in the interior of a conductor material. General gluing methods could be useful to study several problems, among them is the viability of several definitions of a quasi-local mass in general relativity. Can gluing provide a way to define quasi-local mass? Or can gluing provide a way to determine that no such quasi-local mass definition is possible?

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