PROBLEM 1. Negate the following statement: For all ε > 0, there is some δ > 0 such that if |x − 3| < δ, then |x² − 9| < ε.

PROBLEM 2. Express the following using ∀ and ∃: There is some b ∈ ℝ such that the equation x² − b = 0 has no solution.

PROBLEM 3. Let n ∈ ℕ.
(a) Use induction to show that exactly one element of the set {n, n + 1, n + 2, n + 3} is divisible by 4.
(b) Use the Division Lemma to show that exactly one element of the set {n, n + 1, n + 2, n + 3} is divisible by 4.

PROBLEM 4 Let x ∈ ℕ = {1, 2, . . .}.
(a) Prove that x² + x is even.
(b) Prove that (x² + x)/2 is divisible by x if and only if x is odd.
(c) Prove that (x² + x)/2 is divisible by x + 1 if and only if x is even.

PROBLEM 5. (Houston 26.7 (iii)) Show that if x² − 3x + 2 < 0, then 1 < x < 2.

PROBLEM 6. (Houston 27.23 (v)) Prove that every common divisor of a, b ∈ ℤ is a divisor of gcd(a, b).

PROBLEM 7. (a) Calculate gcd(52, 221).
(b) Find m, n ∈ ℤ such that 52m + 221n = gcd(52, 221).

PROBLEM 8. Let n ∈ ℕ. Prove that n is composite if and only if n has a factor a that satisfies

1 < a ≤ √n.