Section 5.1

5.3 Disprove the statement: If \( n \in [1, 2, 3, 4, 5] \), then \( 3 | (2n^2 + 1) \).

5.6 Let \( a, b \in \mathbb{Z} \). Disprove the statement: For every two positive integers \( a \) and \( b \),
\[(a + b)^3 = a^3 + 2a^2b + 2ab + 2ab^2 + b^3.\]

5.8 For positive real numbers \( a \) and \( b \), it can be shown that \((a + b)/(1/a + 1/b) \geq 4\). Does it therefore follow that \((c^2 + d^2)/(1/c^2 + 1/d^2) \geq 4\) for every two positive numbers \( c \) and \( d \)?

Section 5.2

5.11 Prove that there is no smallest positive irrational number.

5.19 Prove that \( \sqrt{3} \) is irrational. [Hint: First prove for an integer \( a \) that \( 3 | a^2 \) if and only if \( 3 | a \). Recall that every integer can be written as \( 3q, 3q + 1, \text{ or } 3q + 2 \) for some integer \( q \).]

5.31 Use a proof by contradiction to prove the following. Let \( m \in \mathbb{Z} \). If \( 3 \nmid (m^2 - 1) \), then \( 3 | m \).