1. Compute the sup, inf, max and min (whenever these exist) for the following sets.

   a. \( S_1 = \left\{ 1 + \frac{1}{n} \mid n \in \mathbb{N} \setminus \{0\} \right\} \)
   
   b. \( S_2 = (-3, -1] \cup [1, 2) \cup \{7\} \)
   
   c. \( S_3 = (-3, -1] \cup [1, 2) \cup \{-4\} \)
   
   d. \( S_4 = \{y \mid y = x^2 - 9, \text{ and } x \in \mathbb{R}\} \)
   
   e. \( S_5 = \{x \mid x^2 - 9 < 0, \text{ and } x \in \mathbb{R}\} \)

2. Is it possible for a subset of \( \mathbb{R} \) to have a maximum, but no supremum? If yes, give an example. If no, provide a brief justification.

3. Prove Beck Proposition 8.43.


5. Prove Beck Proposition 8.45. \textit{Hint: Use Proposition 8.50 twice.}