28.6 (iii): Suppose \( x = qy + r \), where \( x, y, q, r \in \mathbb{Z} \) (you do not need to assume that \( y \) does not divide \( x \)). We want to prove that \( \gcd(x, y) = \gcd(y, r) \). Let \( d := \gcd(x, y) \). This means that \( d \) divides \( x \) and \( y \). By the relation \( r = x - qy \), it follows from Theorem 27.5 that \( d \) divides \( r \). So \( d \) is a common divisor of \( y \) and \( r \), and hence \( d \leq \gcd(y, r) \).

Next, we want to prove that \( d \) is the greatest integer that divides \( y \) and \( r \). For this, we follow Houston’s suggestion to prove by contradiction (however, there is a direct approach which also works). Then we assume there is some \( e \) which divides \( y \) and \( r \), and \( d < e \). Then \( e \) also divides \( x = qy + r \), and so \( e \leq \gcd(x, y) = d \), which is a contradiction.

Here is an alternative way to finish the proof after the first paragraph: By definition, \( \gcd(y, r) \) divides \( y \) and \( r \), so it must divide \( x = qy + r \) by Theorem 27.5. This implies \( \gcd(y, r) \leq d = \gcd(x, y) \), which proves \( \gcd(x, y) = \gcd(y, r) \).

28.19 (i): \( \gcd(14592, 6468) = 12 \) and \( \gcd(-12870, 4914) = 234 \).