1. Prove that if $ab \mid ac$, then $b \mid c$, where $a, b, c \in \mathbb{Z}$ and $a \neq 0$.

Proof: If $ab \mid ac$, then $ac = abk$ for some $k \in \mathbb{Z}$. Since $a \neq 0$, we can divide both sides by $a$ to get $c = bk$, which is exactly the statement that $b$ divides $c$.

2. Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.

Proof: If $a \mid b$ and $b \mid c$, then we can write $b = ak$ and $c = bl$ for some $k, l \in \mathbb{Z}$. Then $c = bl = akl$, and so $a$ divides $c$.

3. Verify that 101 is prime, but try not to do too much work. Hint: Prove by contradiction. Argue that if this is not true, then there is a prime that divides 101, but this prime is not very big (how big?). Then rule out cases.

Proof: For sake of contradiction, assume 101 is not prime. Then there is some $a \in \mathbb{Z}$ such that $a \mid 101$ and $1 < a \leq \sqrt{101} < 11$. We may assume that $a$ is prime, so $a \in \{2, 3, 5, 7\}$. However, none of these numbers divides 101:

\[
101/2 = 50 + 1/2, \quad 101/3 = 33 + 2/3, \quad 101/5 = 20 + 1/5, \quad 101/7 = 14 + 3/7.
\]

This is a contradiction.