1. (a) (10 points) Use a power series to define \( \sin(x) \), where \( x \) is a real number measured in degrees. Briefly explain your answer.

(b) (10 points) Define \( \sin(x) \) using the power series definition, with \( x \) measured in degrees. Determine the value of

\[
\lim_{x \to 0} \frac{\sin(x)}{x}.
\]

Briefly explain your answer.
2. (30 points) Define \( \sin(x) \) and \( \cos(x) \) using the unit circle definition, with \( x \) measured in radians. Use the limit definition of derivative, and the identities

\[
\lim_{x \to 0} \frac{\sin(x)}{x} = 1 \\
\lim_{x \to 0} \frac{1 - \cos(x)}{x} = 0
\]

(1)

to show that

\[
\frac{d}{dx} \sin(x) = \cos(x).
\]
3. (20 points) Define \( \sin(x) \) using the unit circle definition, with \( x \) measured in radians. Does \( \sin : \mathbb{R} \rightarrow \mathbb{R} \) have an inverse? Explain your answer.
4. (30 points) Let $\Delta$ be an isosceles triangle. Recall that this means $\Delta$ has two sides $a, b$ with the same length. Let $0 \leq \theta \leq \pi$ be the measure of the angle between the two sides $a, b$. Find the value of $\theta$ that maximizes the area of $\Delta$. You may assume the common length of $a, b$ remains fixed and equal to 1.
Extra Credit. (20 points) Define $\sin(x)$ and $\cos(x)$ using the unit circle definition, with $x$ measured in radians. Show that

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1.$$