Let $n$ be a positive integer, and let $U_n$ denote the set of solutions $z \in \mathbb{C}$ to the equations $z^n = 1$.

1. Show that $U_n$ has $n$ elements.
2. Show that $U_n$ is a group.
3. Which group is $U_n$? (It is a common one.)
4. Write $U_n = \{z_1, z_2, \ldots, z_n\}$. Show that 
   \[ \sum_{j=1}^{n} z_j = 0. \]
5. Find all solutions $z \in \mathbb{C}$ to the equation
   \[ z^5 - 5z^4 + 10z^3 - 10z^2 + 5z - 2 = 0. \]
   Hint: Expand $(z - 1)^5$ and compare with the above.
6. Let $a, b \in \mathbb{C}$. Find all solutions $z \in \mathbb{C}$ to the equation
   \[ (z - a)^n - b = 0. \]
   Note: This is a higher degree generalization of the vertex form of a quadratic.
7. Can every degree $n$ monic polynomial be written in the form $(z - a)^n - b = 0$ for some $a, b \in \mathbb{C}$?