Instructions

1. You will be given exactly 80 minutes for this exam.

2. Nothing on your desk or lap but a pen/pencil, this booklet, and one sheet of notes.

3. There is additional scratch paper at the end of the exam.

4. Show your reasoning and calculations. Unsupported answers may not receive full credit.

5. Cross out (or erase) any statement you do not want graded.

6. Do not open this booklet until you are instructed to do so.
1. (10 points; James) Consider the region between the $x$-axis, and the graph of $y = \sqrt{x}$ for $0 \leq x \leq 3$. Find the volume of the solid obtained by rotating this region around the line $x = -3$. 
2. (10 points each) Evaluate the following integrals.

- (Taylor) \( \int x \tan^{-1}(x) \, dx \)

- (Aaron) \( \int \frac{\sec^4(2t)}{\tan^3(2t)} \, dt \)
• (Jacob W.) \[ \int \frac{2x + 3}{(x + 6)(x^2 + 3x + 2)} \, dx \]

• (Sakthi) \[ \int_{\sqrt{\pi/2}}^{\pi} \frac{\theta^3 \cos(\theta^2)}{\sqrt{\pi/2}} \, d\theta \]
3. (10 points; Zach) Consider the curve described in polar coordinates by

\[ r(\theta) = 108 - \cos(8\theta). \]

Find the equation of the tangent line at \( \theta = \pi/16 \). Hint: You need to compute \( \frac{dy}{dx} \) to find the slope.

Write \( y \) and \( x \) in terms of \( r \) and \( \theta \), then recall that \( \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} \).
4. (10 points) Choose one of the following problems to complete. You do not need to do both.

Option 1 (Raymond): Consider the curve described in polar coordinates by

\[ r(\theta) = \cos(\theta). \]

Find the arc length from \( \theta = 0 \) to \( \theta = \pi \).

Option 2 (Gustavo): Find the length of the curve described by \( y = 4x^{3/2} \) for \( 0 \leq x \leq 4 \).
5. (10 points; Jonathan Se.) Use the trapezoid rule with 5 intervals to estimate \( \int_0^2 x^2 + 1 \, dx \).
6. (20 points) Use the $\epsilon$-$\delta$ definition of limit to prove

$$\lim_{x \to 2} 2x - 4 = 0.$$
7. (10 points each) Determine whether the integral converges or diverges.

- (Shannon) \[ \int_1^{\infty} \frac{1}{(5x^2 + 1)^{2/3}} \, dx \]

- \[ \int_{-\infty}^{\infty} \cos(x)e^{-x^2} \, dx \]
8. (5 points each) Evaluate each of the following limits.

- (Devinda) \( \lim_{x \to 3} \frac{\sqrt{x} - \sqrt{3}}{x - 3} \).

- \( \lim_{x \to 0^+} \sin(x) \ln(\tan(x)) \)
9. (10 points each) Determine whether each series converges absolutely, converges conditionally, or diverges.

- (Jacob) \( \sum_{n=0}^{\infty} \frac{3 + \cos(n)}{e^n} \)

- (Casey) \( \sum_{n=0}^{\infty} \frac{n}{n + 1} \)
• (Marisa) \[ \sum_{n=1}^{\infty} \frac{(-1)^n 2^n n}{(n+1)!} \]

• the Taylor series with center \( c = 1 \) of \( \ln(x) \), evaluated at \( x = 2 \)
10. (10 points; Danielle) Find the first 3 non-zero terms in the Maclaurin series of $\sqrt{x + 1}$. 
11. (a) (10 points) Find the Maclaurin series for \( \frac{x^4}{8-x^7} \).

(b) (10 points) Determine the interval of convergence of the power series in part (a).
12. Extra Credit (10 points) Find all solutions $y$ to the differential equation

$$y'' + y' + y = 0.$$
13. Extra Credit (10 points; Brian) Evaluate

\[ \int (\ln(\sin(x)) + x \cot(x)) (\sin(x))^x \, dx. \]
SCRATCH PAPER