4 Using The Derivative

4.1 Local Maxima and Minima

*** Local Maxima and Minima**

Suppose p is a point in the domain of f:

- f has a **local minimum** at p if $f(p)$ is *less than or equal to* the values of f for points near p.
- f has a **local maximum** at p if $f(p)$ is *greater than or equal to* the values of f for points near p.

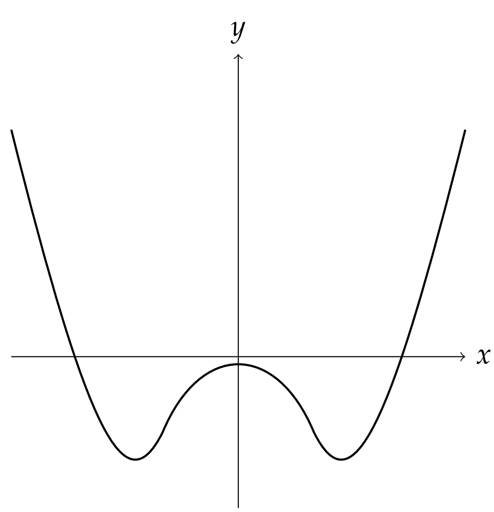
*** How Do We Detect a Local Maximum or Minimum?**

Suppose p is a point in the domain of f:

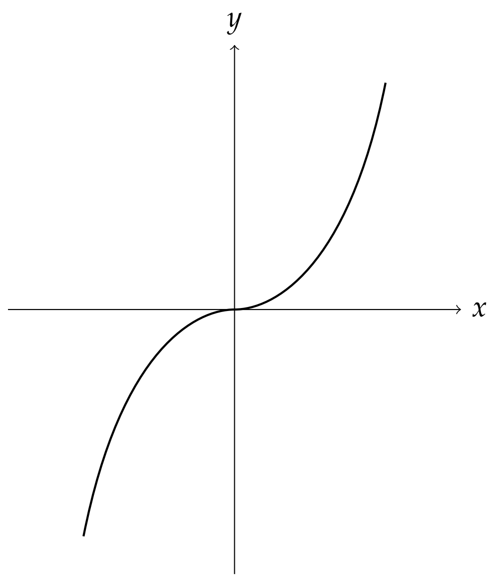
For any function f, a point p in the domain of f where $f'(p) = 0$ or $f'(p)$ is undefined is called a **critical point** of the function.

If a function, continuous on an interval (its domain), has a local maximum or minimum at p, then p is a critical point or an endpoint of the interval.

Example 1 *For $f(x)$ given below, indicate all critical points of the function f. How many critical points are there? Identify each critical point as a local maximum, a local minimum, or neither.*



Example 2 *For $f(x)$ given below, indicate all critical points of the function f. How many critical points are there? Identify each critical point as a local maximum, a local minimum, or neither.*



Example 3 (a) *Graph a function with two local minima and one local maximum.*

(b) *Graph a function with two critical points. One of these critical points should be a local minimum, and the other should be neither a local maximum nor a local minimum.*

*** Testing For Local Maxima and Minima**

First Derivative Test for Local Maxima and Minima
Suppose p is a critical point of a continuous function f. Then, as we go from left to right:

- If f changes from decreasing to increasing at p, then f has a local minimum at p.
- If f changes from increasing to decreasing at p, then f has a local maximum at p.

Example 4 *(a) Graph a function f with the following properties:*

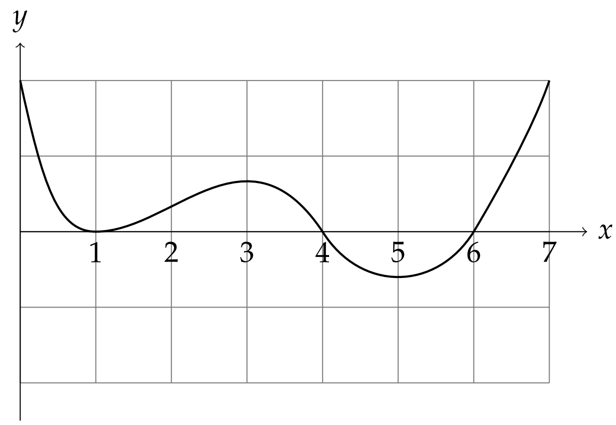
- *$f(x)$ has critical points at $x = 2$ and $x = 5$;*
- *$f'(x)$ is positive to the left of 2 and positive to the right of 5;*
- *$f'(x)$ is negative between 2 and 5.*

(b) Identify the critical points as local maxima, local minima, or neither.

Example 5 *Given the graph of $f'(x)$ below.*

(a) *What are the critical points of the function $f(x)$?*

(b) *Identify each critical point as a local maximum, a local minimum, or neither.*



Example 6 *The derivative of $f(t)$ is given by $f'(t) = t^3 - 6t^2 + 8t$ for $0 \leq t \leq 5$. Graph $f'(t)$, and describe how the function $f(t)$ changes over the interval $t = 0$ to $t = 5$. When is $f(t)$ increasing and when is it decreasing? Where does $f(t)$ have a local maximum and where does it have a local minimum?*

Example 7 *Suppose f has a continuous derivative whose values are given in the following table.*

x	0	1	2	3	4	5	6	7	8	9	10
$f'(x)$	5	2	1	-2	-5	-3	-1	2	3	1	-1

(a) *Estimate the x-coordinates of critical points of f for $0 \leq x \leq 10$.*

(b) *For each critical point, indicate if it is a local maximum of f, local minimum, or neither.*

Second Derivative Test for Local Maxima and Minima
Suppose p is a critical point of a continuous function f, and $f'(p) = 0$.

- If f is concave up at p, then f has a local minimum at p.
- If f is concave down at p, then f has a local maximum at p.

Example 8 *Given $f(x) = x^3 - 9x^2 - 48x + 52$.*

(a) *Find all critical points of f.*

(b) *Use the second derivative test to classify each critical point as a local max, a local min, or neither.*

Example 9 *Find and classify the critical points of* $f(x) = x^3(1-x)^4$ *as local maxima and minima.*

Example 10 *Find constants a and b so that* $f(x) = a(x - b\ln x)$ *has a local minimum at the point* $(2, 5)$.

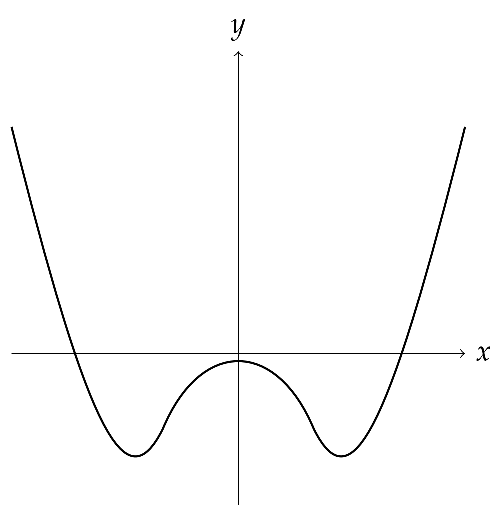
4.2 Inflection Points

*** Concavity and Inflection Points**

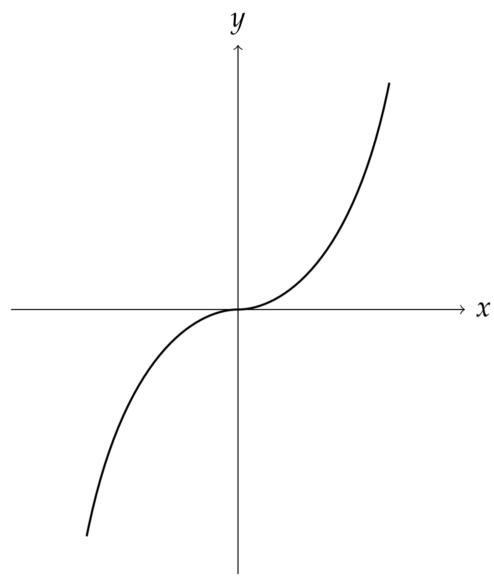
A point at which the graph of a function f changes concavity is called an **inflection point** of f.

If p is an inflection point of f, then either $f''(p) = 0$ or f'' is undefined at p.

Example 1 *For the graph of $f(x)$ given below, indicate the approximate locations of all inflection points. How many inflection points are there?*



Example 2 *For the graph of $f(x)$ given below, indicate the approximate locations of all inflection points. How many inflection points are there?*



Example 3 *Find the inflection points of $f(x) = x^3 - 9x^2 - 48x + 52$.*

Example 4 *Graph a function f with the following properties: f has a critical point at $x = 4$ and an inflection point at $x = 8$; the value of f' is negative to the left of 4 and positive to the right of 4; the value of f'' is positive to the left of 8 and negative to the right of 8.*

Example 5 *Find the critical points and inflection points of $f(x) = xe^{-x}$.*

Example 6 *Find the critical points and inflection points of $f(x) = 2x^3 + 3x^2 - 36x + 5$.*