Given a quadratic polynomial of the form $ax^2 + bx + c$, one can always factor it as

$$ax^2 + bx + c = a(x - r_1)(x - r_2)$$

where the roots $\{r_1, r_2\}$ are determined by the quadratic formula

$$r_{1,2} = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

Some prefer to write a quadratic polynomial in the form $ax^2 + 2bx + c$, in which case the quadratic formula takes the simpler form

$$r_{1,2} = \frac{b \pm \sqrt{b^2 - ac}}{a}$$

However, quadratic polynomials are often factored by inspection, especially in problems where the numbers are chosen carefully to give simple answers. When the leading coefficient $a = 1$, such as $x^2 + 7x + 12$, by inspection

$$x^2 + 7x + 12 = (x + 4)(x + 3)$$

You needed two numbers whose sum is 7 and whose product is 12

A useful trick in factoring by inspection trinomials of the form $ax^2 + bx + c$ is multiplying and dividing by $a$. Then

$$ax^2 + bx + c = ((ax)^2 + b(ax) + ac)/a$$

Now factor the numerator as in the previous case with leading coefficient 1, thinking of $ax$ as your variable.

For example,

$$6x^2 + 13x - 5 = ((6x)^2 + 13(6x) - 30)/6 =$$

(Now we need two numbers whose sum is 13 and product is $-30$, and 15 and $-2$ work.)

$$(6x - 2)(6x + 15)(1/6) = \left(\frac{6x - 2}{2}\right)\left(\frac{6x + 15}{3}\right) = (3x - 1)(2x + 5)$$

Here are some problems to practice. There is no reason to give the answers for these problems. You can multiply your factors and make sure you have the original expression. Nevertheless, we will provide some answers at the end just for the fun of it.
Factor:

1. $x^2 + 6x + 9$
2. $x^2 + 14x + 49$
3. $4a^2 + 4a + 1$
4. $36a^2 + 12ab + b^2$
5. $4x^2 - 12x + 9$
6. $25b^2 - 90b + 81$
7. $9y^2 - 24y + 16$
8. $4x^{2n} - 20x^n + 25$
9. $(x - y)^2 + 4(x - y) + 4$
10. $x^2 - 2x(a + b) + (a + b)^2$
11. $x^2 + 10x + 21$
12. $25b^2 + 5b - 12$
13. $(x - y)^2 + x - y - 2$
14. $(x + y)^2 + 2(x + y) + 1$
15. $1 - 2(x - y) + (x - y)^2$
16. $2x^2 + 5x + 3$
17. $2x^2 + x - 3$
18. $2x^2 - x - 3$
19. $2x^2 - 5x + 3$
20. $10a^2 + 29a + 21$
21. $6x^2 - 7xy + 2y^2$

22. $4b^2 - 17b + 4$

23. $6x^4 - 17x^3y + 10x^2y^2$

24. $5x^n + 20x^{2n} + 20x^{3n}$

25. $12a^{2n} - 7a^n - 12$

26. $21x^3 - 14x^2y - 56xy^2$

27. $-63a^2b^2 + 42ab^2 - 7b^2$

28. $2(x + y)^2 + 5(x + y) + 2$

29. $x^2 + 2ax + (a + b)(a - b)$

30. $ax^2 + (a^2 + a)x + a^2$

Some answers:

4. $36a^2 + 12ab + b^2 = b^2 + (12a)b + 36a^2 = (b + 6a)^2$

6. $25b^2 - 90b + 81 = (1/25)((25b)^2 - 90(25b) + 9 \cdot 5 \cdot 5) = (5b - 9)^2$

23. $6x^4 - 17x^3y + 10x^2y^2 = x^2(1/6)((6x)^2 - 17y(6x) + 60y^2) = x^2(6x - 12y)(6x - 5y)$

(We need two numbers that add to $-17y$ and multiply to $60y^2$; $-12y$ and $-5y$ work.)
24. \(5x^n + 20x^{2n} + 20x^{3n} = 5x^n(4x^{2n} + 4x^n + 1) =\)

\[5x^n(1/4)((4x^n)^2 + 4(4x^n) + 4) =\]

\[5x^n(1/4)((4x^n) + 2)^2 =\]

\[5x^n (1 + 2x^n)^2\]

25. \((3a^n - 4)(4a^n + 3)\)

26. \(7(x - 2y)(3x + 4y)\)

28. \((2 + x + y)(1 + 2x + 2y)\)

29. \((x + a - b)(x + a + b)\)

30. \(a(x + 1)(x + a)\)