If you start with an equation involving \( y \) and \( x \), implicit differentiation lets you solve for \( y' \) in terms of \( y \) and \( x \) without explicitly solving for \( y \) in terms of \( x \).

You use the same method in every problem. You differentiate both sides of the equation with respect to \( x \), and when you encounter the variable \( y \) in an expression, you differentiate with respect to \( y \) and then multiply by \( \frac{dy}{dx} = y' \). The next part is purely algebraic. You get all terms involving \( y' \) on one side of the equation and the other terms on the other side. You then factor out \( y' \) and divide both sides with the remaining factor so that \( y' \) appears alone on one side of the equation. You then have solved for \( y' \) in terms of \( y \) and \( x \).

Examples:

1. Find \( y' \), given \( x^2y - xy^2 + x^3 + y^2 = 0 \).

   Answer:
   
   \[
   \frac{d}{dx}(x^2y) - \frac{d}{dx}(xy^2) + \frac{d}{dx}(x^3) + \frac{d}{dx}(y^2) = 0
   \]
   \[
   x^2 \frac{d}{dx}(y) + y \frac{d}{dx}(x^2) - x \frac{d}{dx}(y^2) - y^2 \frac{d}{dx}(x) + \frac{d}{dx}(x^3) + \frac{d}{dx}(y^2) = 0
   \]
   
   \[
   x^2y' + 2xy - x(2yy') - y^2 + 3x^2 + 2yy' = 0
   \]

   The terms \( 2xy, \ -y^2, \ \) and \( 3x^2 \) do not involve \( y' \) so move them to the other side:

   \[
   x^2y' - x(2yy') + 2yy' = -2xy + y^2 - 3x^2
   \]

   Now factor \( y' \) from all terms on the left side:

   \[
   y'(x^2 - 2xy + 2y) = -2xy + y^2 - 3x^2
   \]

   Now divide both side by the factor \( (x^2 - 2xy + 2y) \) to get

   \[
   y' = \frac{-2xy + y^2 - 3x^2}{x^2 - 2xy + 2y}
   \]

   The answers to problems 2 and 3 are on the next pages

2. Find \( y' \), given \( xy + x^2y^2 = 0 \).

3. Find \( y' \), given \( x^3y^2 + xy^2 + yx^3 + xy^2 = 0 \).
\[ \frac{d}{dx}(xy + x^2 y^2) = 0. \]

\[ xy' + y + 2x^2 yy' + 2xy^2 = 0 \]

\[ xy' + 2x^2 yy' = -(y + 2xy^2) \]

\[ y'(x + 2x^2 y) = -(y + 2xy^2) \]

\[ y' = -\frac{y + 2xy^2}{x + 2x^2 y} \]
\[
\frac{d}{dx} (x^3y^2 + xy^2 + yx^3 + xy^2) = 0.
\]

\[2x^3yy' + 3x^2y^2 + 2xyy' + y^2 + 3yx^2 + y'x^3 + 2xyy' + y^2 = 0\]

\[2x^3yy' + 2xyy' + y'x^3 + 2xyy' = -(3x^2y^2 + y^2 + 3yx^2 + y^2)\]

\[y'(2x^3y + 2xy + x^3 + 2xy) = -(3x^2y^2 + y^2 + 3yx^2 + y^2)\]

\[y' = -\frac{3x^2y^2 + y^2 + 3yx^2 + y^2}{2x^3y + 2xy + x^3 + 2xy}\]