A typical question on an exam asks you to find the derivative of a given function by using the definition. This means you need to compute

\[
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

Can you guess what the most common error is? Click here for the answer.

For each example, find the derivative of \(f(x)\) using the definition. If you can work all these problems without peeking at the solutions, you will most likely be well prepared for this topic on an exam. To check your answer, click inside the blue rectangle.

1. \(f(x) = x^2\)

Answer:

\[
\frac{f(x + h) - f(x)}{h} = \frac{(x + h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{2xh + h^2}{h} = 2x + h \to 2x \text{ as } h \to 0.
\]

2. \(f(x) = x^3 + 2x\)

3. \(f(x) = \frac{1}{x}\)

Answer:

\[
\frac{f(x + h) - f(x)}{h} = \frac{\frac{1}{x + h} - \frac{1}{x}}{h} = \frac{x - (x+h)}{x(x+h)h} = \frac{-h}{x(x+h)h} = \frac{-1}{x(x+h)} \to -\frac{1}{x^2} \text{ as } h \to 0.
\]

4. \(f(x) = \frac{1}{2x + 1}\)

5. \(f(x) = \sqrt{x}\)

Answer:

\[
\frac{f(x + h) - f(x)}{h} = \frac{\sqrt{x + h} - \sqrt{x}}{h} = \left(\frac{\sqrt{x + h} - \sqrt{x}}{h}\right) \left(\frac{\sqrt{x + h} + \sqrt{x}}{\sqrt{x + h} + \sqrt{x}}\right) = \frac{x + h - x}{h(\sqrt{x + h} + \sqrt{x})} = -\frac{1}{\sqrt{x + h} + \sqrt{x}} \to -\frac{1}{2\sqrt{x}} \text{ as } h \to 0.
\]
6. \( f(x) = \sqrt{2x + 1} \)

7. \( f(x) = \frac{1}{\sqrt{x}} \)