Homework for MTH 310
Fall 2017

Textbook:
Abstract algebra, an introduction, by Thomas W. Hungerford, Third edition

Homework for week 08/30-09/01:
Due Wednesday 09/06

- 08/30:
  1)Prove the following statements:

  (i) For every integer x, if x is even, then for every integer y, xy is even.

  (ii) For every integer x and y, if x is odd and y is odd then $x + y$ is even.

  (iii) For every odd integer n, n^3 is odd.

  2)Form the negation of each statement in question 1)

  3)Find out whether the statement:

  $$\forall x \in \mathbb{Z},\ \big((\exists y \in \mathbb{Z},\ \text{such that } x = 3y + 1) \Longrightarrow \big(\exists y \in \mathbb{Z},\ \text{such that } x^2 = 3y + 1\big)\big)$$

  is true or false, then prove it.

  4)Prove that the statement:

  $$\forall x \in \mathbb{Z},\ \big(x \text{ odd } \implies \exists y \in \mathbb{Z}, \text{such that } x^2 = 8y + 1\big)$$

  is true.

- 09/01: Chap 1.1, ex 1,2 and 5 + set-theory exercises:
  5) Explain why if $A \subset B$ and $B \subset C$ then $A \subset C$.

  6) Show that $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

Homework for week 09/06-09/08:
Due Monday 09/11

- 09/06: Chapter 1.2: ex 1abcdef,2,4,8

- 09/08: Chapter 1.3: ex 1,5,15,19,20

Quiz on Friday 09/08
Covers Logic and Proof/Set theory + Chapter 1.1 and 1.2

Homework for week 09/11-09/15:
Due Monday 09/18

- Chapter 2.1: ex 1,5,12

- Chapter 2.2: ex 2,4,8, Chap 2.1: ex 15

- Chapter 2.3: ex 4,8,12

Homework for week 09/18-09/22:
Due Monday 09/25

- 09/18: Chapter 3.1: ex 2,11,17

- 09/20: Chapter 3.1: ex 6,9, Chapter 3.2: ex 8

- 09/22: Chapter 3.2: ex 12,20,32

Quiz on Friday 09/22
Covers Chapter 1.3, 2.1, 2.2, 2.3

Homework for week 09/25-09/29:
Due Monday 10/02

- 09/25: Chapter 3.2: ex 30,36, Chapter 3.3: ex 8,12

- 09/27: Chapter 3.3: ex 24,35,36

- 09/29: Chapter 3.3: ex 10,32,38

Homework for week 10/02-10/06:
Due Monday 10/09

- 10/02: Chapter 3.1: ex 38, Chap 3.2: ex 40, Chap 3.3: ex 29, Chap 4.1: ex 6

- 10/04: Chapter 4.1: ex 2,4,10, Chap 1.2: ex 30, Chap 1.3: ex 32

Quiz on Friday 10/06
Covers Chapter 3.1, 3.2 and 3.3

Exam 1 on Wednesday 10/11
Covers Chapter 1, 2 and 3
List of suggested review exercises for each chapter:
Chap 1.1: ex 11
Chap 1.2: ex 12,14
Chap 1.3: ex 7,27,30
Chap 2.1: ex 10,18
Chap 2.2: ex 9
Chap 2.3: ex 16,18
Chap 3.1: ex 7,16
Chap 3.2: ex 3,6,13
Chap 3.3: ex 11,27,31

Homework for week 10/13-10/20:
Due Monday 10/23

- 10/13: Chapter 4.1: ex 19, Chapter 4.2: ex 2,4
- 10/16: Chapter 4.2: ex 5,14, Chapter 4.3: ex 2
- 10/18: Chapter 4.3: ex 12,20, Chapter 4.4: ex 2,4
- 10/20: Chapter 4.4: ex 10,16,24

Homework for week 10/23-10/27:
Due Monday 10/30

- 10/23: Chapter 4.5: ex 2,4,5
- 10/25: Chapter 4.5: ex 6,8,16,18
- 10/27: Chapter 4.6: ex 2,4

Quiz on Friday 10/27
Covers Chapter 4.1 to 4.4

Homework for week 10/30-11/03:
Due Monday 11/06

- 10/30: Chapter 4.6: ex 1(b),3(b), Chapter 5.1 ex 1,2
- 11/01: Chapter 5.1: ex 4,6,8,10

Exam on Wednesday 11/08
Covers Chapter 4 and 5.1
List of suggested review exercises:
Chapter 4.1: ex 5,7,11,13
Chapter 4.2: ex 3,7,10,15,16
Chapter 4.3: ex 3,4,11,20
Chapter 4.4: ex 1,3,9
Chapter 4.5: ex 1,13,19
Chapter 4.6: ex 1,2,3,4
Chapter 5.1: ex 3,5,11

Homework for week 11/10-11/17:
Due Monday 11/20

- 11/10: Chapter 5.2: ex 2,6,14
- 11/13: Chapter 5.3: ex 2,4,6
- 11/15: Chapter 5.3: ex 8,10
- 11/17: Chapter 6.1: ex 4,6,10

Homework for week 11/20-11/22:
Due Monday 11/27

- 11/20: Chapter 6.1: ex 8,16,20,24,26

- 11/22: Chapter 6.1: ex 22,36

Homework for week 11/27-12/01:
Due Monday 12/04

- 11/27: Chapter 6.2: ex 4,8,10,12

- 11/29: Chapter 6.2: ex 22,27, Chapter 6.3: ex 8

- 12/01: Chapter 6.3: ex 4,6,10

Quiz on Friday 12/01
Covers Chapters 5.2, 5.3, 6.1, 6.2

Final Exam on Tuesday 12/12
Covers Chapter 1 to 6.
Important notions:
Chapter 1: Divisibility, gcd, the fundamental theorem of arithmetics.
Chapter 2: Definition of congruence mod n and congruence classes. Modular arithmetic: computing sums, products, powers of elements of $\mathbb{Z}_n$. Structure of $\mathbb{Z}_n$: units, zero divisors, $\mathbb{Z}_p$ field iff p prime.
Chapter 3: Definition of rings. Showing that a subset is a subring. Definition of units, zero divisors, homomorphisms, isomorphisms. Showing that a map is an homomorphism. Obstructions for two rings being isomorphic (commutativity, characteristic, number of units/zero divisors...).
Chapter 4: Long polynomial division. Euclidian algorithm for gcd in $F[x]$. Arithmetics in $F[x]$. Remainder theorem. Rational root test, Eisenstein criterion. Irreducible polynomials in $\mathbb{R}[x]$ and $\mathbb{C}[x]$.
Chapter 5: Congruence mod $p(x)$ and congruence classes. Computations in rings $F[x]/(p(x))$. Structure of $F[x]/(p(x))$: units, $F[x]/(p(x))$ field iff $p(x)$ irreducible.
Chapter 6: Definition of ideals; checking that a subset of a ring is an ideal. Quotient rings. Kernel of a morphism. 1st isomorphism theorem. Prime and maximal ideals, definition and characterization of their quotient rings.
List of suggested review exercises (some we have already seen, others are new):
Chapter 1.3: ex 7,17
Chapter 2.1: ex 5,6
Chapter 2.3: ex 2,4
Chapter 3.1: ex 16,22,24
Chapter 3.2: ex 3,32
Chapter 3.3: ex 21,22
Chapter 4.1: ex 5
Chapter 4.2: ex 5
Chapter 4.3: ex 10,13,22,23
Chapter 4.4: ex 8,11,12,25
Chapter 4.5: ex 5,14,18

Chapter 4.6: ex 5
Chapter 5.1: ex 1,3
Chapter 5.2: ex 1,2,3,4,8,9
Chapter 5.3: ex 1,5,11
Chapter 6.1: ex 37,40,44,45
Chapter 6.2: ex 13,21,23
Chapter 6.3: ex 2,14,15,16,19,20