You may work together on solving these problems, but what you hand in must be your work written in your own words.

1. (5 points) If $\vec{r}$ is a twice-differentiable vector-valued function, show that

$$\frac{d}{dt} [\vec{r}(t) \times \vec{r}'(t)] = \vec{r}(t) \times \vec{r}''(t)$$

2. (5 points) If $\vec{r}(t) \neq 0$, show that

$$\frac{d}{dt} |\vec{r}(t)| = \frac{1}{|\vec{r}(t)|} \vec{r}(t) \cdot \vec{r}'(t)$$

*Hint:* One way is to write $|\vec{r}(t)|^2 = \vec{r}(t) \cdot \vec{r}(t)$ and differentiate both sides with the chain rule.
3. (5 points) Consider the function \( \vec{r}(t) = \langle t, \ln t, t \rangle, \; t > 0. \)

   (a) Compute the curvature \( \kappa \) of \( \vec{r} \).

   (b) What is \( \lim_{t \to \infty} \kappa(t) \)? Describe this geometrically.

4. (5 points) A particle moves along the helix \( x(t) = 3 \cos t, \; y(t) = 3 \sin t, \; z(t) = t \), starting at time 0 (at the point \( (3, 0, 0) \)). How long does it take for the particle to travel 1 unit?