Do not start this exam until instructed; you will have 90 minutes to finish the exam. No notes, books, calculators, phones or electronic devices are allowed on this exam. If you have a question, raise your hand; otherwise, there is no talking during the exam.

There are 14 problems on this exam on 6 pages, in addition to this cover page. The point values of each problem vary, but are listed in the questions.

Good luck!

From SMBC.
**Fill in the Blanks Section.** No work needed, and no partial credit available. \[4+4+6+4+4+3=25\]

1. (4 points) A vector normal to the plane \(3x + y = 7z\) is \(\vec{n} = \) ____________________.

2. (4 points) A surface is given by \(F(x, y, z) = x^3 + 2yz + y^2\). The equation of the tangent plane at the point \((1, 0, 1)\) is given by ____________________.

3. (2+2+2=6 points) A particle has acceleration \(\vec{a}\), velocity \(\vec{v}\) and position \(\vec{r}\). You are given that

\[
\vec{a}(t) = \vec{i} - 3\vec{j} \\
\vec{v}(0) = \vec{k} \\
\vec{r}(0) = 2\vec{j} + \vec{k}
\]

Find the following:

(a) \(\vec{v}(t) = \) ____________________

(b) \(\vec{r}(t) = \) ____________________

(c) Does the particle go through the origin? ________

Extra Work Space.
4. (4 points) Suppose that \((3, 4)\) is a critical point for the surface \(h(x, y)\), and say that

\[ h_{xx}(3, 4) = 6, \quad h_{yy} = 1, \quad h_{xy}(3, 4) = -2 \]

Choose one of the following:

(a) \((3, 4)\) is a local maximum of \(h\).
(b) \((3, 4)\) is a local minimum of \(h\).
(c) \((3, 4)\) is a saddle point of \(h\).
(d) There is not enough information to determine this.

5. (4 points) The two legs of a right triangle are measured to be 2 cm and 4 cm with a possible error of at most 0.3 cm in each. Use differentials to estimate the maximum error in the calculated value of the area of the triangle: \(\ldots\) cm².

6. (3 points) Consider \(\vec{a} = \langle 1, 0, 0 \rangle\) and \(\vec{b} = \langle 4, 5, -1 \rangle\). Then the vector projection \(\text{proj}_{\vec{a}} \vec{b}\) is \(\ldots\).
**Standard Response Questions.** Show all work to receive credit. \([10 + 15 + 10 + 5 + 10 + 10 + 10 + 5 = 75]\]

7. (10 points) Find a value of \(a\) such that \(u(x, t) = \sin(4t) \cos(ax)\) satisfies the differential equation \(u_{tt} = 4u_{xx} \).  

8. (5+10=15 points) Evaluate the following limits. If one or both does not exist, say so.

(a)  
\[
\lim_{(x,y) \to (-1,3)} \frac{2xy}{x^2 + y^2}
\]

(b)  
\[
\lim_{(x,y) \to (0,0)} \frac{2xy}{x^2 + y^2}
\]
9. \(7+3=10\) points Consider \(\vec{r}(t) = (2\sin t, t, 2\cos t)\).

(a) Find the arc length function for \(\vec{r}(t)\) starting from the point \((0, 0, 2)\).

(b) Suppose you move 1 unit along \(\vec{r}(t)\) in the positive direction. Where are you now?

10. (5 points) Find a vector normal to the plane passing through the points \(P = (1, 2, 3), Q = (1, 0, 0),\) and \(R = (2, 2, 2)\).
11. (10 points) Find the linearization of \( f(x, y) = 2 + \sqrt{1 + x + \sin y} \) at the point \((0, \pi)\).

12. (10 points) Find the partial derivative \( \frac{\partial T}{\partial r} \) for

\[
T = \frac{v}{u}, \quad u = \frac{2rq^2}{s^2}, \quad v = rs
\]

Your final answer should include only the variables \( q, r, s \).
13. (5+5=10 points) Consider the function \( g(x, y, z) = x + \ln(yz) \).

(a) Find \( \nabla g \) at the point \((3, 1, 2)\).

(b) Find the directional derivative of \( g \) at \((2, 1, 2)\) in the direction of \( \vec{i} + \vec{k} \).

14. (5 points) A ball is thrown in the air at an angle of 45° and an initial speed of 10\( \sqrt{2} \) m/s. How far away does the ball hit the ground? (Ignore air resistance, and use the value of \( g \approx 10 \text{ m/s}^2 \)).