Do not start this exam until instructed; you will have 50 minutes to finish the exam. No notes, books, calculators, phones or electronic devices are allowed on this exam. If you have a question, raise your hand; otherwise, there is no talking during the exam.

There are 12 problems on this exam on 6 pages, in addition to this cover page. The point values of each problem vary, but are listed in the questions.

Good luck!

From Argyle Sweater.
1. (2+2+2+2=8 points) For the following problems, no work is necessary - just give the answer.

(a) Describe the teaching sequence for area.

\[ \text{Count square units} \]
\[ \text{Rectangles w/ whole number sides.} \]
\[ \text{Fractional sides.} \]
\[ \text{Real sides.} \]

(b) Name at least 2 polygons that tessellate.

Squares

(Any) triangles

(c) 3 pairs of congruent sides proves congruence between triangles (SSS congruence).
If, similarly, there are 4 pairs of congruent sides between two quadrilaterals are the two figures necessarily congruent?

No. (See rhombus.)

(d) If a square and a rectangle that is not a square have the same perimeter which has the larger area?

*Hint: Construct an example.*

\[ \text{The square.} \]
\[ \text{5} \]
\[ \text{5 vs. 9.9} \]

2. (2+2+2+2=8 points) For the following problems, mark true or false. No work is necessary.
All parallelograms with a right angle are rectangles. True  False
A triangle with sides of lengths 24, 35, 51 units is a right triangle. True  False
A parallelogram with perpendicular diagonals is necessarily a kite. True  False
Every number can be written as a fraction. True  False
3. (20 points) State and prove the Pythagorean Theorem.

Theorem: If \( a \) and \( b \) are the lengths of the legs of a right triangle and \( c \) is the length of the hypotenuse, then \( a^2 + b^2 = c^2 \).

1) \( \Box\text{QRS} \) is a square.

Proof: It's a rhombus. Now \( y + (90 - y) + z = 180 \)

by angles on line (let P) gives \( y = 90 \).

A rhombus with a right angle is a square.

2) Area \( \Box\text{QRS} \) = \( c^2 \).

3) Area of big square = \( \begin{cases} (a+b)^2 & \text{side lengths} \\ c^2 + 4 \cdot \frac{1}{2} ab & \text{\Box\text{QRS and } 4 \text{ triangles}} \end{cases} \)

4) Compute: \( (a+b)^2 = c^2 + 4 \cdot \frac{1}{2} ab \)

\[ a^2 + b^2 + 2ab = c^2 + 2ab \]

\[ a^2 + b^2 = c^2. \]
4. (5+7=12 points) Prove the area of a triangle is \( \frac{1}{2} \text{base} \times \text{height} \) in the following cases:

(a) A right triangle

Complete to a rectangle. The two triangles are congruent by \( SSS \). The rectangle has area \( bh \), so each has \( \frac{1}{2} \) of that area.

(b) Altitude is on the interior

Cut into right triangles. The total area is

\[
\frac{1}{2}bh + \frac{1}{2}(b-x)h
\]

\[
= \frac{1}{2}bh - \frac{1}{2}xh
\]

\[
= \frac{1}{2}bh.
\]

5. (10 points) Find the area of an equilateral triangle with sides of length 1 unit.

Cut as shown. Want height \( h \):

\[
\frac{\sqrt{3}}{2}
\]

by properties of a 30-60-90 \( \Delta \).

\[
\therefore \quad h = \frac{\sqrt{3}}{2}
\]

\[
\therefore \quad \text{Area} = \frac{1}{2} \cdot 1 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}.
\]
6. (5 points) Explain how to find the area of the following parallelogram in 2 different ways. Mark any lengths used in your explanation.

1) Move this triangle.
   Get a rectangle. Area $= bh$.

2) Cut to two triangles.
   Each has area $\frac{1}{2}bh$.
   Total $= 2 \cdot \frac{1}{2}bh$.

7. (5 points) Which of the triangles in the following figure are congruent?

- Triangle $\triangle ABE$ and $\triangle DCE$ are congruent by $\angle$SAA.
- Triangle $\triangle ABC$ is not congruent to either.

8. (3+2=5 points) Suppose you know the longest side of a triangle is 17 cm and one leg is 8 cm long.
   (a) Explain why the area cannot be found.

   Here are two such triangles with different areas:

   (b) Find the area if the triangle is a right triangle.

   $\frac{1}{2} \cdot (8 \text{ cm}) \cdot (15 \text{ cm}) = 4 \text{ cm} \cdot 15 \text{ cm} = 60 \text{ cm}^2$
9. (5 points) Find the angle $x$.

\[ 12 - 7 = 5 \quad \text{by Pythagorean Theorem} \]

10. (6 points) Find the area of the shaded region. All lengths are given in inches.

\[
\text{Area} = \text{Area (rectangle)} - \text{Area (holes)}
\]
\[
= (3\text{ in} \times 4\text{ in}) - [(1\text{ in}) \times (1\text{ in}) + (1\text{ in}) \times (1\text{ in})]
\]
\[
= 12\text{ in}^2 - 2\text{ in}^2 = 10\text{ in}^2
\]

Alternate Solution:
\[
\text{Get } 4\text{ in}^2 + 3\text{ in}^2 + 2\text{ in}^2 = 9\text{ in}^2
\]
11. (8 points) In the figure, \( AP = AQ \) and \( BP = CQ \). Show that \( BQ = CP \).

\[
\begin{align*}
\text{Note} & \quad AB = AP + PB \\
& = AQ + QC \\
& = AC, \text{ which is why the sides marked III are congruent.}
\end{align*}
\]

\[
\text{Now } BQ = CP \text{ since the segments correspond.}
\]

12. (8 points) In the figure, \( AB = AC \) and \( AB \) is parallel to \( HK \). Show that \( HK = HC \).

\[
\text{Recall: Isosceles} \quad \frac{+}{\text{Two equal sides}} \quad \frac{\text{Two equal angles}}{\quad}.
\]