Name ______________________

PID ______________________

Practice Exam 1

MTH 310, Thursday June 7, 2018

Instructions: This exam is closed notes, closed books, no calculators and no electronic devices of any kind. There are four problems worth 25 points each. If a problem has multiple parts, it may be possible to solve a later part without solving the previous parts. Solutions should be written neatly and in a logically organized manner. Partial credit will be given if the student demonstrates an understanding of the problem and presents some steps leading to the solution. Correct answers with *no work* will be given *no credit*. The back sheets may be used as scratch paper but will not be graded for credit.

1	2	3	4	Total

Problem 1.

a. (10 points) Find the multiplicative inverse of [3] in $\mathbb{Z}_{100}$.

b. (10 points) Find two distinct solutions to $x^2 \equiv 1 \mod 12$ in $\mathbb{Z}_{12}$.

Problem 2.

a. (10 pts) Write the fraction $q = \frac{7}{24}$ in the form $q = \frac{k}{8} + \frac{l}{3}$ for some $k, l \in \mathbb{Z}$.

b. (15 pts) Let $a, b \in \mathbb{Z}$ and suppose the gcd of $(a, b) = 1$.

Prove that if $r = \frac{n}{ab}$ for $n \in \mathbb{Z}$ then there exist $k, l \in \mathbb{Z}$ such that $r = \frac{k}{a} + \frac{l}{b}$.

Problem 3. Let R be a ring and $I \subset R$ be a subring. I is called an *left ideal* if whenever $a \in I$ and $r \in R$ then $ra \in I$.

a. (5 pts) Show that the zero ring $\{0_R\}$ is a left ideal of R. (Do not prove the subring part.)

b. (5 pts) Show that $\mathbb{Z}$ is not a left ideal of $\mathbb{Q}$. (Do not prove the subring part.)

c. (15 pts) Let $\phi : R \to S$ be a surjective ring homomorphism and suppose $I \subset R$ is a right ideal. Show that the image of I defined as $\phi(I) := \{b \in S : s = \phi(a) \text{ for some } a \in I\}$ is a right ideal of S.

Problem 4.

a. (10 pts) Write the definition for a ring R to be an *integral domain*.

b. (15 pts) Let R be a commutative ring with identity. Suppose that for all $a, b, c \in R$ with $a \neq 0_R$; if $ab = ac$ then $b = c$. Prove that R is an integral domain.