MTH 310: HW 4

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Due: June 11, 2018

1. **(Hungerford 4.1.3)** List all the polynomials of degree 3 in $\mathbb{Z}_2[x]$.
2. **(Hungerford 4.1.11)** Show that $1 + 3x$ is a unit in $\mathbb{Z}_9[x]$.
3. **(Hungerford 4.1.16)** Let R be a commutative ring with identity and $a \in R$. If $1_R + ax$ is a unit in $R[x]$, show that $a^n = 0_R$ for some integer $n > 0$.
4. **(Hungerford 4.1.20)** Let $D : \mathbb{R}[x] \to \mathbb{R}[x]$ be the derivative map defined by
$$D(a_0 + a_1x + a_2x^2 + \cdots + a_nx^n) = a_1 + 2a_2x + \cdots + na_nx^{n-1}.$$
Prove that D is not a homomorphism of rings.
5. **(Hungerford 4.2.4)** Let F be a field and $f(x), g(x) \in F[x]$. If $f(x)|g(x)$ and $g(x)|f(x)$ show that $f(x) = cg(x)$ for some nonzero $c \in F$.
6. **(Hungerford 4.2.5)**
   (a) Let $f(x) = x^4 + 3x^3 + 2x + 4$ and $g(x) = x^2 - 1$ in $\mathbb{Z}_5[x]$. Show that $g(x)|f(x)$.
   (b) Let $f(x) = x^4 + x + 1$ and $g(x) = x^2 + x + 1$ in $\mathbb{Z}_2[x]$. Adapt the Euclidean Algorithm for integers to find the gcd of $(f(x), g(x))$.
7. **(Hungerford 4.2.15)** Let F be a field and $f(x), g(x), h(x) \in F[x]$. Prove that if $h(x)|f(x)$ and gcd of $(f(x), g(x)) = 1$ then gcd of $(h(x), g(x)) = 1$.
8. **(EC–worth .5% of final grade)** Let R be a commutative ring and let $f(x), g(x) \in R[x]$ with $f(x)$ nonzero. Prove that if $f(x)g(x) = 0_R$ then there exists $c \in R$ such that $cg(x) = 0_R$.