Name: Solutions ______________________ **Section**: ____________

Clear your desk of everything excepts pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. Find the most general anti-derivative.
   (a) (1 point) $f(x) = 2x^3 + 1$ on $\mathbb{R}$
   $F(x) = \frac{2}{4}x^4 + x + C$.

   (b) (1 point) $f(x) = \sin x + 10 \sec x \tan x$ on $(-\frac{\pi}{2}, \frac{\pi}{2})$
   $F(x) = -\cos x + 10 \sec x + C$.

2. (3 points) Estimate the value of $\sqrt{37}$ by using Newton's method for $f(x) = x^2 - 37$ and $x_1 = 6$.
   Find the values of x_2.
   $f'(x) = 2x$
   $$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 6 - \frac{36-37}{12} = \underline{6 + \frac{1}{12}}.$$

3. (0 points) There are cases when Newton's method does not work. Can you explain why it fails to estimate the root of the equation $x^3 - 3x + 6 = 0$ if $x_1 = 1$.

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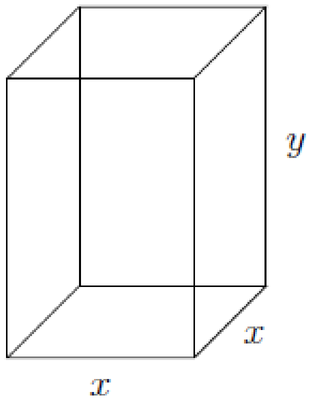
4. Suppose you want to build a tank in the shape of a rectangular prism with a square bottom and **no top** using exactly 40 ft^2 of material. See the figure below. In this problem you will find the dimensions that maximize the volume of the tank.

   (a) (1 point) Find the volume $V(x, y)$ of the tank in terms of x and y
   $V(x, y) = x^2y$.

   (b) (1 point) Find an equation relating x and y to the total amount of materials 40 ft^2.
   Total Surface area $= 40 = x^2 + 4xy$.

   (c) (1 point) Find the volume $V(x)$ as a function of only x for $x > 0$.
   Use (b) to solve for y then back substitute into (a).
   $y = \dfrac{40 - x^2}{4x}$.
   $V(x) = x^2\left(\dfrac{40 - x^2}{4x}\right) = \underline{\dfrac{x(40 - x^2)}{4}}$.

   (d) (2 points) Find the values of x and y that maximize the volume.
   Domain: $0 < x < \infty$.
   Critical points: $V'(x) = \dfrac{1}{4}(40 - 3x^2) = 0 \implies x = \pm\sqrt{\dfrac{40}{3}}$. $-\sqrt{40/3}$ is not in the domain.
   The only crit. pt. is $\underline{x = \sqrt{40/3}}$.
   The y-value is found as in part (c): $\underline{y = (40 - (40/3))/(4\sqrt{40/3})}$.



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