Name: Solutions ______________________ **Section**: ____________

Clear your desk of everything excepts pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. Let $f(x) = \dfrac{\sqrt{2x^4 - x - 3}}{x^2 - 1}$.

   (a) (3 points) Find all horizontal asymptotes for $f(x)$.
   Your answer should be of the form of a line $y = L$ for some number L.

   Compute the limits at infinity:

$$\lim_{x\to\infty} \frac{\sqrt{2x^4 - x - 3}}{x^2 - 1} \cdot \frac{\sqrt{\frac{1}{x^4}}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\sqrt{2 - \frac{1}{x^3} - \frac{3}{x^4}}}{1 - \frac{1}{x^2}} = 2$$

$$\lim_{x\to-\infty} \frac{\sqrt{2x^4 - x - 3}}{x^2 - 1} \cdot \frac{-\sqrt{\frac{1}{x^4}}}{\frac{1}{x^2}} = -\lim_{x\to-\infty} \frac{\sqrt{2 - \frac{1}{x^3} - \frac{3}{x^4}}}{1 - \frac{1}{x^2}} = -2.$$

   Therefore, the horizontal asymptotes are $y = 2$ and $y = -2$.

   (b) (2 points) Find all vertical asymptotes for $f(x)$.
   Your answer should be of the form: $x = c$ for some number c.

   Set the denominator equal to 0, $x^2 - 1 = 0 \implies x = -1, 1$. Since the numerator is not zero at either of these points, that means they are both vertical asymptotes.

Continue on to back side

2. Let $f(x) = \frac{3x^2}{x^2-4}$, $f'(x) = \frac{-24x}{(x^2-4)^2}$, $f''(x) = \frac{24(3x^2+4)}{(x^2-4)^3}$.

(a) (1 point) Find the domain and critical points of $f(x)$.

Domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

$f'(-2)$, $f'(2)$ do not exist, but those values are not in the domain.
Set $f'(x) = \frac{-24x}{(x^2-4)^2} = 0 \implies x = 0$.
Critical points: $x = 0$.

(b) (2 points) Identify the intervals over which $f(x)$ is increasing and decreasing.

We need to test points in the following intervals: $(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$.
$f'(-3) = \frac{24*3}{5^2} > 0$
$f'(-1) = \frac{24}{9} > 0$
$f'(1) = \frac{-24}{9} < 0$
$f'(3) = \frac{-24*3}{5^2} < 0$.
By the increasing/decreasing test:
$f(x)$ is decreasing on $(0,2)$ and $(2,\infty)$.
$f(x)$ is increasing on $(-\infty,-2)$ and $(-2,2)$.

(c) (2 points) Identify the intervals over which $f(x)$ is concave up and concave down.

Set $f''(x) = \frac{24(3x^2+4)}{(x^2-4)^3} = 0$, there are no solutions. There are no inflection points.
Test the sign of $f''(x)$ on the intervals $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$.
$f''(-3) = \frac{24(3^3+4)}{5^3} > 0$
$f''(0) = \frac{24(4)}{(-4)^3} < 0$
$f''(3) = \frac{24(3^3+4)}{5^3} > 0$
By the concavity test:
$f(x)$ is concave up on $(-\infty,-2)$ and $(2,\infty)$.
$f(x)$ is concave down on $(-2,2)$.

______ / 10