Name: Solutions ______________________ **Section**: ____________

Clear your desk of everything excepts pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (a) (1 point) Find the linearization $L(x)$ of $f(x) = \sqrt{x+14}$ at the point $x = 2$.

   The derivative is $f'(x) = \dfrac{1}{2\sqrt{x+14}}$. The linearization at $x = 2$ is

   $$\begin{aligned} L(x) &= f'(2)(x-2) + f(2) \\ &= \frac{1}{8}(x-2) + 4. \end{aligned}$$

   (b) (1 point) Use you answer in part (a) to find a linear approximation of $\sqrt{17}$.

   Linear approximation: if x is close to 2, then $f(x) \approx L(x)$.
   Set $f(x) = \sqrt{x+14} = \sqrt{17}$ to find that $x = 3$, that is $\sqrt{17} = f(3)$. Therefore,

   $$L(3) = \frac{1}{8}(3-2) + 4 = \frac{1}{8} + 4 \approx \sqrt{17}.$$

   (Check the error: $\sqrt{17} = 4.1231\ldots$ compared to $4 + \frac{1}{8} = 4.125$. The error $|\sqrt{17} - 4.125| < .02$.)

2. Let $f(x) = \dfrac{4}{x} + x$.

   (a) (2 points) Find all critical points of $f(x)$ on the interval $(1, 3)$.

   Domain of $f(x)$: $(-\infty, 0) \cup (0, \infty)$.
   $f'(x) = -\dfrac{4}{x^2} + 1 = 0 \implies x^2 = 4 \implies x = \pm 2.$
   $f'(0)$ does not exists, but since 0 is not in the domain it is not a critical point.
   The only critical point on the interval $(1, 3)$ is at $\underline{x = 2}$.

   (b) (2 points) Find the absolute maximum and absolute minimum values of $f(x)$ on the interval $[1, 3]$.

   $f(x)$ at the critical point: $f(2) = \frac{4}{2} + 2 = 4$.
   $f(x)$ at the endpoints: $f(1) = \frac{4}{1} + 1 = 5$ and $f(3) = \frac{4}{3} + 3$.
   Compare all values: $f(2) < f(3) < f(1)$.
   The absolute minimum is $f(2) = 4$ and absolute maximum is $f(1) = 5$.

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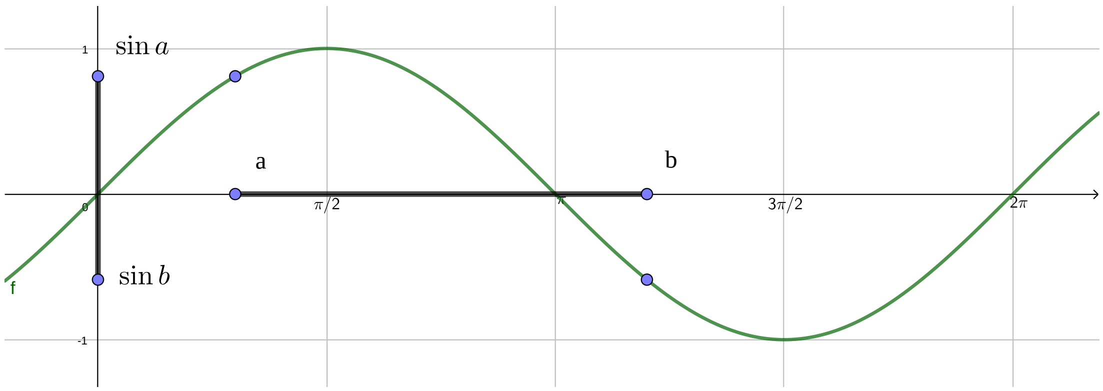
3. (a) (1 point) Verify that $f(x) = \sin(x)$ satisfies the hypothesis of the Mean Value Theorem.

$\sin(x)$ is continuous on $\underline{\quad(-\infty, \infty)\quad}$

$\sin(x)$ is differentiable on $\underline{\quad(-\infty, \infty)\quad}$.



(b) (3 points) Apply the Mean Value Theorem to $f(x) = \sin(x)$ on the interval $[a, b]$ to show that

$$|\sin b - \sin a| \leq |b - a|.$$

(Hint: $f'(x) = \cos(x) \leq 1$ for all x.) *In the figure, the line segment on the $y-$axis has length $|\sin b - \sin a|$, while the line segment on the $x-$axis has length $|b - a|$.*

Notice that $f'(x) = \cos x$. By the M.V.T. there is a point c in (a, b) such that

$$\frac{\sin b - \sin a}{b - a} = \cos c.$$

Take the absolute value on both sides:

$$\left|\frac{\sin b - \sin a}{b - a}\right| = |\cos c| \leq 1.$$

$$\implies \quad \left|\frac{\sin b - \sin a}{b - a}\right| \leq 1.$$

$$\implies \quad |\sin b - \sin a| \leq |b - a|.$$