Name: Solutions ______________________ **Section**: ____________

Clear your desk of everything excepts pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (3 points) Find an equation of the tangent line to the curve $x^2 + y^3 + xy = 1$ at the point $P(2, -1)$.

   Take the implicit derivative

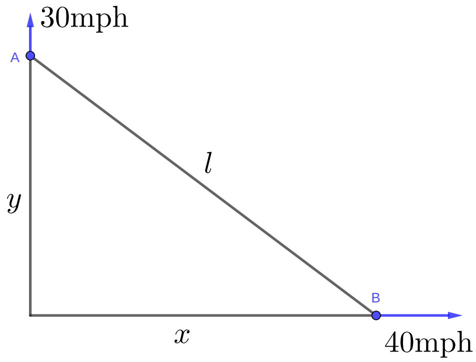
$$\frac{d}{dx}(x^2 + y(x)^3 + xy(x)) = 0$$
$$2x + 3y^2y' + y + xy' = 0$$

   Plug-in the point values $x = 2$ and $y = -1$

$$4 + 3y' - 1 + 2y' = 0 \implies y' = -3/5.$$

   Therefore the tangent line is: $y = -\frac{3}{5}(x-2) - 1$.

2. (3 points) Two cars leave at an intersection. One travels north at 30 mph and the other travels east at 40 mph. How fast is the distance between them increasing after 2 hours?



   Let $x(t)$ be the position of car A, $y(t)$ be the position of car B and $l(t)$ the distance between two cars. Then, its given that $x'(t) = 40$ mph and $y'(t) = 30$ mph. We want $l'(2)$. Since the cars are moving at constant speed we can calculate their future positions: $x(2) = 2 * 40 = 80$ and $y(2) = 2 * 30 = 60$.

   The relation is given by Pythagorean's theorem: $x^2 + y^2 = l^2$. Taking the implicit derivative we get

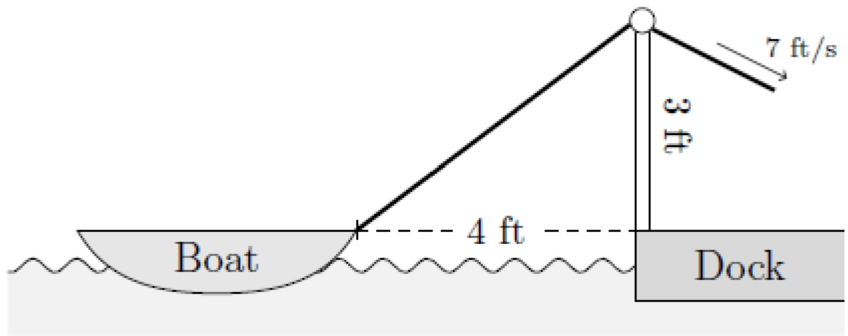
$$2xx' + 2yy' = 2ll'.$$

   Plug-in values and solve so l' we have:

$$l' = \frac{2 * 80 * 40 + 2 * 60 * 30}{2\sqrt{80^2 + 60^2}} \text{mph}.$$

Continue on to back side

3. (4 points) A boat is pulled into a dock by a rope attached to the bow (front end) of the boat and passing through a pulley on the dock that is 3 ft higher than the bow of the boat. If the rope is pulled in at a rate of 7 ft/s, at what speed is the boat approaching the dock when it is 4 ft from the dock?



Let

$$x(t) - \text{boat to dock}$$
$$l(t) - \text{boat to pulley}$$

Its given that $l'(t) = 7$ ft/s. We want $x'|_{x=4}$.

The relation is given by Pythagorean's theorem: $x^2+3^2 = l^2$. Taking the implicit derivative: $2xx' = 2ll'$.

Plug-in values and solve for x':

$$x' = \frac{7\sqrt{4^2+3^2}}{4} = \frac{35}{4}\text{ft/s.}$$