Name: ______________________________ **Section**: 022

Clear your desk of everything excepts pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (3 points) Let $f(x) = \dfrac{\sin(8x)\cos(x)}{x}$ and compute the limit:

$$\lim_{x\to 0} f(x) =$$

$$= \lim_{x\to 0} \frac{\sin(8x)\cos(x)}{x} \qquad \text{(multiply top and bottom by } 8x\text{)}$$

$$= \lim_{x\to 0} \frac{\sin(8x)}{8x}\frac{8x\cos(x)}{x} \qquad (x \text{ cancels on top and bottom})$$

$$= \left(\lim_{x\to 0} \frac{\sin(8x)}{8x}\right)\left(\lim_{x\to 0} 8\cos x\right) \qquad \text{(use limit law)}$$

$$= (1)(8\cos(0)) = \underline{\ \ 8\ \ }.$$

2. (2 points) **Multiple Choice. Circle the best answer. No partial credit available**
Let $h(x) = f(2g(x))$. Suppose that

$$g(1) = 3, \qquad g'(1) = 2, \qquad \text{and} \qquad f'(6) = 1.$$

Determine the value of $h'(1)$.

A. 0
B. 2
C. 4 ✓
D. 6
E. None of the above.

Use the chain rule where the outside function is $f(u)$ and the inside function is $u = 2g(x)$,

$$h'(x) = f'(2g(x)) \cdot 2g'(x).$$

Evaluate the expression at $x = 1$,

$$h'(1) = f'(2g(1)) \cdot 2g'(1) = f'(6) \cdot 2 \cdot 2 = 4.$$

Continue on to back side

3. The height of a projectile (in feet) changing in time (in seconds) is given by the function

$$h(t) = -16t^2 + 64t + 5.$$

(a) (1 point) Find $h'(t)$. *(include units)*

$$h'(t) = -32t + 64 \ \frac{\text{ft}}{\text{s}}.$$

The units come from the formula for the derivative:

$$\frac{dh}{dt} = \lim_{\Delta t \to 0} \frac{\Delta h}{\Delta t} \implies \frac{\text{ft}}{\text{s}}.$$

We can also remember that $h'(t)$ is the velocity and physically represents how height changes with time.

(b) (2 points) Is the projectile moving up or down at $t = 1$?

Since $h'(1) = 32 > 0$ we conclude the velocity is positive and the height is increasing. Thus, the projectile is moving up.

(c) (2 points) What is the maximum height of the projectile? *(include units)*

The maximum height occurs when the velocity is zero, $h'(t) = 0$.

$$0 = -32t + 64 \qquad \implies \qquad t = 2 \text{ secs}.$$

Thus, at $t = 2$ the projectile is at its maximum height

$$\underline{h(2) = -16(2)^2 + 64(2) + 5 \text{ ft}}\ .$$

(You must include units! You do not have to simplify answer.)

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