Name: Solutions ______________________ **Section**: 022

Clear your desk of everything excepts pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (3 points) Let $f(x) = \frac{1}{x}$. Use the definition of the derivative to compute $f'(1)$.

$$f'(1) = \lim_{h\to 0} \frac{\frac{1}{1+h} - \frac{1}{1}}{h} \quad \text{(use common denominator)}$$
$$= \lim_{h\to 0} \frac{1-(1+h)}{(1+h)h} \quad \text{(cancelation: } 1-1=0\text{)}$$
$$= \lim_{h\to 0} -\frac{h}{(1+h)h} \quad \text{(cancelation: } \frac{h}{h}=1\text{)}$$
$$= \lim_{h\to 0} -\frac{1}{1+h} \quad \text{(direct substitute } h=0\text{)}$$
$$= -1$$

2. (3 points) Evaluate the limit

$$\lim_{x\to 25} \frac{x-25}{\sqrt{x}-5}$$

$$= \lim_{x\to 25} \frac{x-25}{\sqrt{x}-5} \cdot \left(\frac{\sqrt{x}+5}{\sqrt{x}+5}\right) \quad \text{(Multiply by conjugate on top/bottom)}$$
$$= \lim_{x\to 25} \frac{(x-25)(\sqrt{x}+5)}{x-25} \quad \text{(Expand the bottom and cancel)}$$
$$= \lim_{x\to 25} \sqrt{x}+5 \quad \text{(direct substitution)}$$
$$= 10$$

Continue on to back side

3. Let $f(x) = 1 - x\sqrt{x}$.

(a) (2 points) Find the derivative function $f'(x)$.

We can use the product rule and power rule.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(1 - x\sqrt{x}) \\ &= 0 - \left((1)\sqrt{x} + x\frac{1}{2\sqrt{x}}\right) \\ &= -(\sqrt{x} + \frac{1}{2}\sqrt{x}) \\ &= -\frac{3}{2}\sqrt{x}. \end{aligned}$$

(b) (2 points) Find the equation of the tangent line to $f(x)$ at the point $(1, 0)$.

The slope of the tangent line is given by $f'(1) = -\frac{3}{2}$. Using the point-slope form for the tangent line we have that

$$-\frac{3}{2} = \frac{y - 0}{x - 1}.$$

Solving for y

$$y = -\frac{3}{2}x + \frac{3}{2}.$$