Name: Solutions ______________________ **Section**: ____________

Clear your desk of everything excepts pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. Find the general indefinite integral

   (a) (2 points) $\int \left(x + \frac{1}{x}\right)^2 dx$

$$\begin{aligned} &= \int x^2 + 2 + \frac{1}{x^2} dx \\ &= \frac{1}{3}x^3 + 2x - \frac{1}{x} + c \end{aligned}$$

   (b) (2 points) $\int \frac{\cos\theta}{(1+\sin\theta)^2} d\theta$

   Let $u = 1 + \sin\theta$ and $du = \cos\theta d\theta$. Then,

$$\begin{aligned} \int \frac{\cos\theta}{(1+\sin\theta)^2} d\theta &= \int \frac{1}{u^2} du \\ &= -\frac{1}{u} + c \\ &= -\frac{1}{1+\sin\theta} + c. \end{aligned}$$

2. (2 points) Find the average value of $f(x) = \sqrt{x}$ on the interval $[0, 4]$.

$$\begin{aligned} f_{ave} &= \frac{1}{4}\int_0^4 \sqrt{x} dx \\ &= \frac{2}{12} x^{3/2}\Big|_0^4 \\ &= \frac{2}{12}(4)^{3/2} = \frac{4}{3}. \end{aligned}$$

Continue on to back side

3. A particle is moving with velocity given by $v(t) = \cos t$ (with units m/s).

(a) (1 point) Given that the initial position is $s(0) = 0$, find the position at time $t = \frac{3\pi}{2}$ seconds.

We can write the above as an initial value problem:

$$\frac{d}{dt}s = \cos t, \qquad s(0) = 0.$$

The general solution is: $s(t) = \sin t + c$.

The initial value is: $s(0) = 0 = \sin 0 + c \implies c = 0$.

Thus, $s(t) = \sin t$ and $s(3\pi/2) = \sin(3\pi/2) = -1$m.

(b) (2 points) What is the total distance traveled on the interval $[0, \frac{3\pi}{2}]$?

We know that $v(t) = \cos t = 0$ when $t = \pi/2$. Thus:

$$\begin{aligned}
\text{Total distance} &= \int_0^{3\pi/2} |\cos t| dt \\
&= \int_0^{\pi/2} \cos t dt - \int_{\pi/2}^{3\pi/2} \cos t dt \\
&= (\sin(\pi/2) - \sin 0) - (\sin(3\pi/2) - \sin(\pi/2)) \\
&= 3 \text{ m} .
\end{aligned}$$

(c) (1 point) Find the average velocity on the interval $[0, \frac{3\pi}{2}]$.

$$\begin{aligned}
v_{ave} &= \frac{2}{3\pi} \int_0^{3\pi/2} \cos t dt \\
&= \frac{2}{3\pi} \sin t \Big|_0^{3\pi/2} \\
&= \frac{2}{3\pi} (\sin(3\pi/2) - \sin 0) \\
&= -\frac{2}{3\pi} \text{ m/s.}
\end{aligned}$$

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